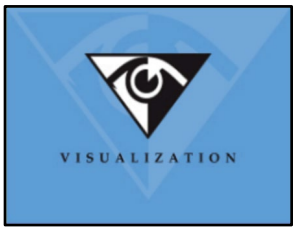


Lesson M3.5
The Great Race

In this lesson, students will use the Visualization Thinking Strategy to explore the different ways to represent fractions on a number line. Students should develop the understanding that when fractions are represented on a number line, each line segment should be equal in length. Students will apply visualization to mentally partition a number line into fractional subunits.

For information about the meaning of *fractions as measures* or fractions on a number line, see the *Math Matters* book (p. 99-101, 109-111)

CCSS.MATH.CONTENT.3.NF.A.2
Understand a fraction as a number on the number line; represent fractions on a number line diagram.

CCSS.MATH.CONTENT.3.NF.A.2.B
Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

Standards for Mathematical Practice
MP4. Model with mathematics
MP5. Use appropriate tools strategically

Time Frame: ~ 60 minutes
To allow students to investigate the concepts in this lesson fully, it may take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the **Explain** section. When restarting the lesson, be sure to start with a brief review of what students discovered during the **Explain** section of the lesson before moving into the **Elaborate/Extend** section.

Materials
Sticky notes
White Board or plastic sheet protectors/paper and markers (class set)
Manipulatives: square inch tiles, unifix cubes, pattern blocks, any other math materials in your classroom that students might use
8 x 11 inch paper (1 piece per student)
Story of the Tortoise and the Hare - http://read.gov/aesop/025.html
"Number of Hurdles" cards (1 per pair)
Open Number Line Template (1 page per student, so each student gets 4 number lines)

"This interactive book is presented by the Library of Congress, adapted from the book "The Aesop for Children: with Pictures by Milo Winter," published by Rand, McNally & Co in 1919. This work is considered to be in the public domain in the United States." (Library of Congress)

Engage

Gather students in a space in the classroom that is conducive to discussions and introduce students to the idea of running a race by using the story "Tortoise and Hare." Read the short passage about the Tortoise and the Hare: Library of Congress Aesop Fables (read.gov). Ask students to visualize what is happening in the story. Provide students with the opportunity to share their thoughts. Ask students to visualize the race course as a number line, with the starting line of the race at the **zero** point and the finish line at the **one** point, thus representing *one whole race distance.*

Tell students that there are other types of races too. Show students the videos of the women's hurdles (https://youtu.be/w-6-SSLDrrk) and the men's hurdles (https://youtu.be/jLHg4OX_hSo) races.* Ask students what they notice about these races and how that may differ from the races they visualized after reading the "Tortoise and the Hare."

*Note that the women's race is 100m and the men's race is 110m. Since when comparing fractions we must start with the same size whole, leave out the distance for the students and have them assume the races are the same distance.

Look for?

- Students who can share their ideas clearly or elaborate on their ideas related to visualization. (communicative)
- Students who ask meaningful questions about representing fractions visually. (curious)

Explore

Continue the discussion by asking students to share what they noticed about the hurdles races. Give students think time and then ask them to share their ideas with a partner. Have multiple students share out what they noticed.

Have students work with a partner to develop a representation (concrete, visual, or symbolic) to show a race that would have **only three hurdles**. Students can use manipulatives, drawings, or other materials they think would be helpful to represent the race. Allow time for students to brainstorm and develop their ideas. As the pairs of students work, observe to see how students are representing the race. Take note of any pairs that are using a linear or number line representation.

Look For

- Students who already are using a number line representation (Perceptive)
- Students who develop representations with different strategies (Strategic)
- Students who apply their learning from the Engage portion of the lesson (Example: Use a strategy their partner mentioned) (Resourceful)

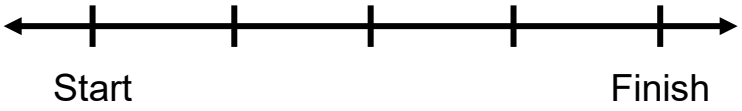
Explain

Come back together as a class and facilitate a discussion about how the students represented the race. You can use a doc cam or "fish bowl" to have students share their ideas. Be sure to have any students who represented the race in a linear way or as a number line share their ideas with the class.

Regardless of if any students used a linear or number line representation, initiate a discussion of how a number line can be used to represent the race with the three hurdles. Draw an open number line on chart paper/white board for all students to see.

Tell students that this is a number line and we can use it as another way to represent the race. Ask students how we could show the three hurdles on the number line. Students ideas might lead to the number line looking like this to show the three hurdles:

Remind students that the three marks show each of the hurdles and ask students what they notice about the spaces between the hurdles. Students should come up with the idea that the spaces between the hurdles are the same size/length. Then ask students if there is anything missing from the representation. Students should identify that the start and the end of the race is not on the representation. Have students explain what that should look like and add their ideas to the representation. The representation should look something like this:



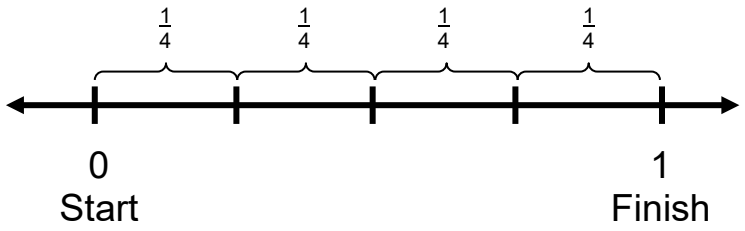
*Some perceptive students might observe that the actual distance in a hurdles race between the starting line and the first hurdle and the last hurdle and the finish line are different from the distances between hurdles. But for the purposes of this task, students should assume that all are the same.

Ask students if this representation looks like anything that they have used in math before. We want students to bring up the idea of a number line here. Continue the discussion of a number line:

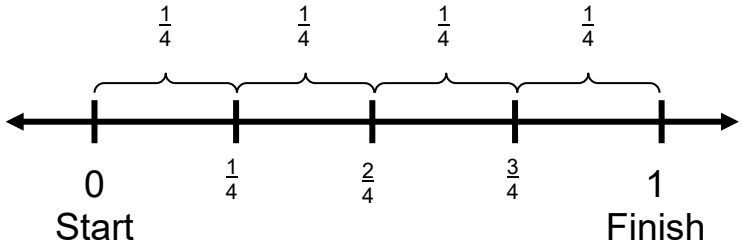
> If from here to here (tracing from Start to Finish on the number line with your finger or a pointer) shows the whole race, how can we label where we put the hurdles using numbers?

Give students think time and then ask them to share their ideas with a partner. Have multiple students share out what their ideas of how to label the number line. Student ideas may include that each line segment shows

one-fourth, that the start would be 0, and that the finish would be 1 . If that comes up you might add to the number line in this way:



Continue using questioning to lead students to label the tick marks (hurdles) with the fractions they represent. Ultimately, the number line should look like this:



Explain to students that we had to partition (divide) the distance from 0 to 1 in equal parts and **emphasize** that the fraction is not the single tick mark that labels it. The fraction is the distance between the tick marks; each line segment represents one-fourth and the tick mark at the end of each unit tells how many one-fourth units are accumulated from zero.

Look For

- Students who restate their peers' explanation in a different way. (communicative)
- Students who ask questions about others' explanations. (curious)
- Students who clearly articulate their thinking. (communicative)
- Students who relate the concept of the number line to real-life situations. (perceptive)

Elaborate/Extend

Target Task

Tell students that they have been asked by the PE teacher to set up a race with hurdles for their class and the PE teacher needs them to create a

*Note that the actual distance between the starting line and the first hurdle and the last hurdle and the finish line are different. But for the purposes of this task, students should assume they are the same.

number line diagram to show the fractional parts of the race if there are different numbers of hurdles. For example, one group might create a number line fraction representation to show what the race would look like if there were 5 hurdles and another group would create one for 6 hurdles. *Be sure students understand that there is a distance between the starting line and the first hurdle and the last hurdle and the finish line, as well as the distances between each of the hurdles (thus, a race with **4** hurdles would have **5** segments. Also tell students that for our purposes, all the distances between the starting line and the first hurdle, between the hurdles, and from the last hurdle to the finish line are the same length.

Assign students to pairs. Provide each pair a set of the "Number of Hurdles" cards and multiple copies of the open number line template. Explain to students that they should shuffle the pile of Number of Hurdles cards and then flip over the top card. When they have flipped their card, they should visualize a track with a starting line, a finish line, and that number of hurdles. Each student in the pair will create a representation of the hurdles race with a starting line, the hurdles, and the finish line. Once each student in the pair is finished they should compare their number line representations and determine how many fractional parts their race diagram/number line is representing. Ask them to label their representations to show the fractions that are represented. Tell each pair they should complete three different race diagrams to share with the PE teacher (they would complete this task 3 times for different numbers of hurdles).

Extend the Task

For students who demonstrated high potential behaviors related to creating the number line representations and/or their understanding of fraction concepts, modify the target task to increase its complexity. To extend the task, tell the students that the PE teacher also wants to know if there are any race diagrams that will have hurdles in the same places (ex. the diagram with 3, 7, and 11 hurdles will result in number lines that represent ¼, 1/8, and 1/12 and would therefore have some equivalent fractions - where the hurdles would be in the same place on the track).

To complete the extended task, students will create three race diagrams/number line representations but give them the sets of cards with 3/7/11 hurdles (fractional parts of 1/4, 1/8, and 1/12) or the cards with 2/5/8/11 hurdles (fractional parts of 1/3, 1/6, 1/9, and 1/12). Students will then work to compare the fractions among the three race diagrams they complete to identify where there would be hurdles in the same places on each. Have students circle these on each number line representation and discuss how they know these are the same places (represent the same amount of the whole).

Scaffolding and Support

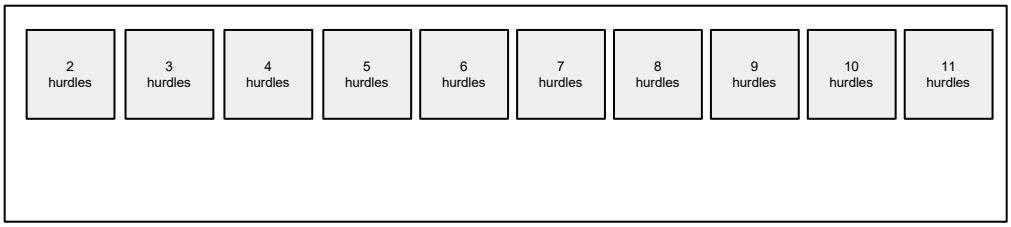
For students who need support or scaffolding to create their fraction number line representations, consider providing cards that have fewer hurdles on them. For example, you can have them do halves (1 hurdle) and fourths (3 hurdles). Students who need support can also be encouraged to use manipulatives to help them make sure the distances between the fractional parts of the number line are equal in length.

Look For

- Students who express their ideas about the number line related to the race, starting line and finish line clearly. (communicative)
- Students who use information from the previous tasks with fourths and eighths to help them with the elaborate task. (resourceful)
- Students who use unique strategies to determine how to equally partition the number line. (creative)
- Students who make connections to equivalent fractions independently. (perceptive)

Evaluate

Label the class white board with the number of hurdles on each card.



Gather students to this part of the classroom and have them tape their race diagrams/number line representations under the corresponding number of hurdles.

Give students time to consider what they notice about the diagrams. Ask them to share some of their observations with a partner. Then, facilitate a discussion about what the students notice about the visual representations on the number lines and record their ideas on a chart paper. Students may make observations such as:

- As the number of hurdles increases, each fractional part of the race decreases.
- As the number of hurdles decreases, each fractional part of the race increases.
- The size of the whole race does not change.

Encourage students who completed the Extend the Task can contribute ideas related to the race diagrams that would have hurdles in the same place. This could be an introduction to equivalent fractions.

Number of Hurdles Cards

2 hurdles	3 hurdles
4 hurdles	5 hurdles
6 hurdles	7 hurdles

8 hurdles	9 hurdles
10 hurdles	11 hurdles