Lesson M2.7
Is it half?

In this lesson, students will explore the idea that one-half of two identical wholes have to be the same size, but not necessarily the same shape. The Encapsulation Thinking Skill is exemplified in the discussion parts of the lesson, especially during the Explain and Evaluate sections of the lesson where students are discussing their ideas. Students should precisely explain their reasoning related to the idea that halves of wholes of the same size do not have to be the same shape.

For more information related to fractions as part of a whole, see the *Math Matters* book (p. 99-105 & 114-119).

CCSS.MATH.CONTENT.2.G.A.3
Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. **Recognize that equal shares of identical wholes need not have the same shape.**

Standards for Mathematical Practice
MP3. Construct viable arguments and critique the reasoning of others.

MP5. Use appropriate tools strategically.

Time Frame: ~ 60 minutes
To allow students to investigate the tasks and concepts in this lesson fully, it will likely take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered the day before during the Explain section of the lesson.

Materials
Square sticky notes (3 inch squares)
Shapes to investigate (attached)
Square inch tiles
Chart paper/markers
Tape

Engage

Pose the question, "What does one-half mean?" Allow students think time, and then ask them to share with a partner sitting next to them. Listen in to ideas that the students are sharing with one another and note ideas that should be shared with the whole class.

Upon bringing the class back together, have groups share out ideas about what one-half means. Allow for multiple interpretations and ask for examples. Record students' responses on chart paper or white board. Some ideas students might share are:

- Half means two equal pieces of a whole
- Half is breaking a shape in two pieces
- The two pieces have to be the same size

If students do not bring up the idea that the two pieces of a whole need to be the same size for one of them to represent one-half, be sure to ask questions to lead students to this idea before moving on with the lesson.

Explore

Provide each student with one square sticky note (3 in. square). Ask students how they could show one-half *of the sticky note*. Allow students time to do so. Students might fold or cut the sticky note in half vertically, horizontally, or diagonally. They also might color in half of the sticky note. If a student only draws a vertical, horizontal, or diagonal line, be sure to ask them how they could prove that each piece is the same size. As students complete this task for one sticky note, provide another sticky note and ask if the student can determine another way to show one-half.

Look For

- Students who are precise in making the two pieces equal in size and shape. (Strategic)
- Students who are able to model one-half in more than one way. (Creative)

Explain

Bring the class back together and have students share how they showed one-half with their sticky notes. As students share, have them tape their own examples to a chart. They should add their examples to other student examples that are similar (ex. those that folded or cut diagonally in the same group).

Students are engaged in the Encapsulation thinking strategy during this discussion because they are starting to think about what it means to divide a shape in half.

Some questions that could be asked during this part of the lesson are:

- *How does this model show one-half?*
- *What do we notice about all of the examples in this group?* (point to the examples in one of the groupings) Emphasize that students should be specific in supporting their ideas.

The discussion should conclude with students' understanding the following points. Encourage several students to articulate and repeat these points.

- One-half of the square sticky notes can be shown in multiple ways,
- Each of the rectangles that result from folding/cutting the sticky note in half vertically or horizontally represents one-half, as does each of the triangles that result when folding/cutting it in half diagonally, and
- The two rectangles are the same size and the two triangles are the same size.

Look For

- Students who are able to make connections between different examples. (Perceptive)
- Students who support their ideas by referring back to the examples. (Communicative)
- Students who articulate clear explanations. (Communicative)
- Students who begin to ask additional questions related to making halves and/or other fractional parts of a whole (Curious)

Elaborate/Extend

Tell students that when they were explaining their ideas about showing one-half of the square you noticed that sometimes one-half was shown with a rectangle shape and sometimes with a triangle shape.

Target Task

Show students the two squares attached below and ask them how it could be that both of them are cut in half when the shapes look different. Ask students to share ideas about how they could investigate and prove that the line in each shape divides it in half. Also, have students brainstorm materials they could use to investigate the problem (ex. students could use square inch tiles, sticky notes, cut the shapes, etc.).

Allow student pairs time to investigate the problem. As pairs are investigating, observe students' ideas and work. Make note of the ways students are investigating and look for evidence of understanding and high-potential behaviors.

Extend the Task

For students who were observed engaging in one of the high-potential behaviors or who demonstrated advanced understanding of the concept in the Explain section of the lesson, this target task can be extended by asking students to investigate how different shapes can all represent one-fourth of the shape instead of one-half (see the Shapes for Extend the Task below). Have students investigate how at least two of the different shapes can each represent one-fourth of the whole square.

Scaffolding and Support

If pairs seem to need more support, consider suggesting that they try using the materials in a specific way. For example:

- Students could use the square inch tiles to cover each of the halves and estimate how many whole square cover each piece to prove that they are the same size.
- Students could cover the rectangle half with the triangle half, cut off the extra part of the triangle and use the piece that was cut off to cover the rest of the rectangle. This can be done by covering the triangle half with the rectangle half as well.
- Students could use sticky notes to cover the rectangle and triangle half pieces. This would help them see that the triangle and rectangle half pieces are two square sticky notes or one square and two half-squares (triangles) and are therefore the same size or area.

Be sure when suggesting materials for students to try that students still try to figure out how the materials could be used without being told directly. For example, consider asking "How might you use the square inch tiles to help you show the triangle and rectangle halves are the same size?"

Look For

- Students who continue to persevere through the task even if at first they are unsuccessful in solving it. (Resilient)
- Students who investigate the task in multiple or original ways. (Creative)
- Students who apply strategies from previous lessons/parts of this lesson to investigate this task in different ways. (Strategic)

Evaluate

As pairs are wrapping up their investigation, bring the class back together. Begin the discussion by asking each pair that investigated the shapes that represented one-half how they investigated the problem and what they learned about representing one-half. Some questions that could be asked during this discussion are:

- *How do the rectangle and triangle shape both represent one-half?*
- *Why is it important that we recognize that one-half can be represented in different ways?*

Sample student responses may include:

- They both are half of the square. (Ask students to elaborate on this idea, to prove why.)
- I use the same number of squares/triangles to cover each of them. (Ask students to show an example of this)
- I can cut the rectangle (or triangle) to make it fit on the triangle (or rectangle). (Ask students to show an example of this.)

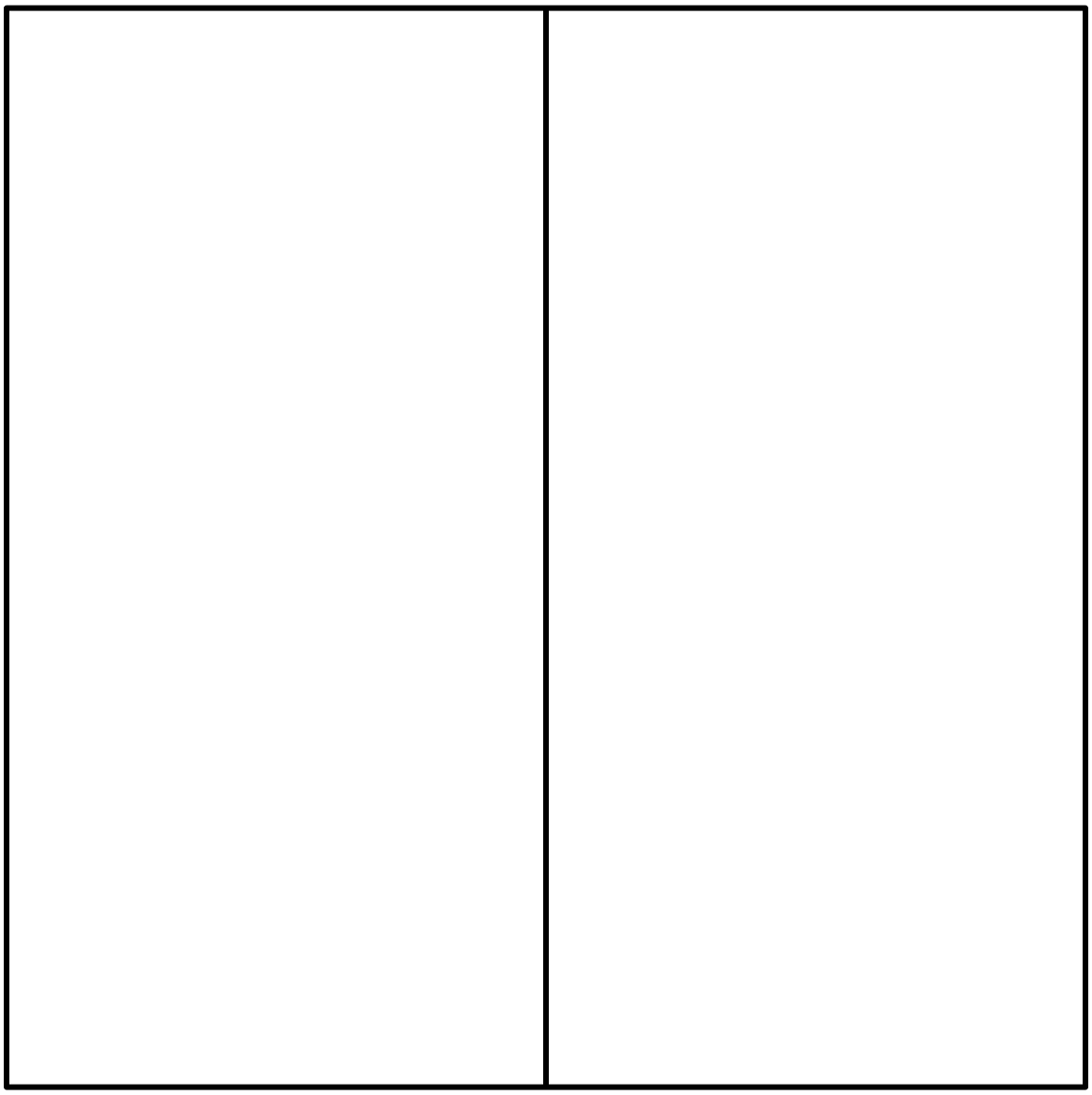
This part of the discussion engages students in Encapsulation because students have to articulate that the different shapes can still represent one-half because they are the same size.

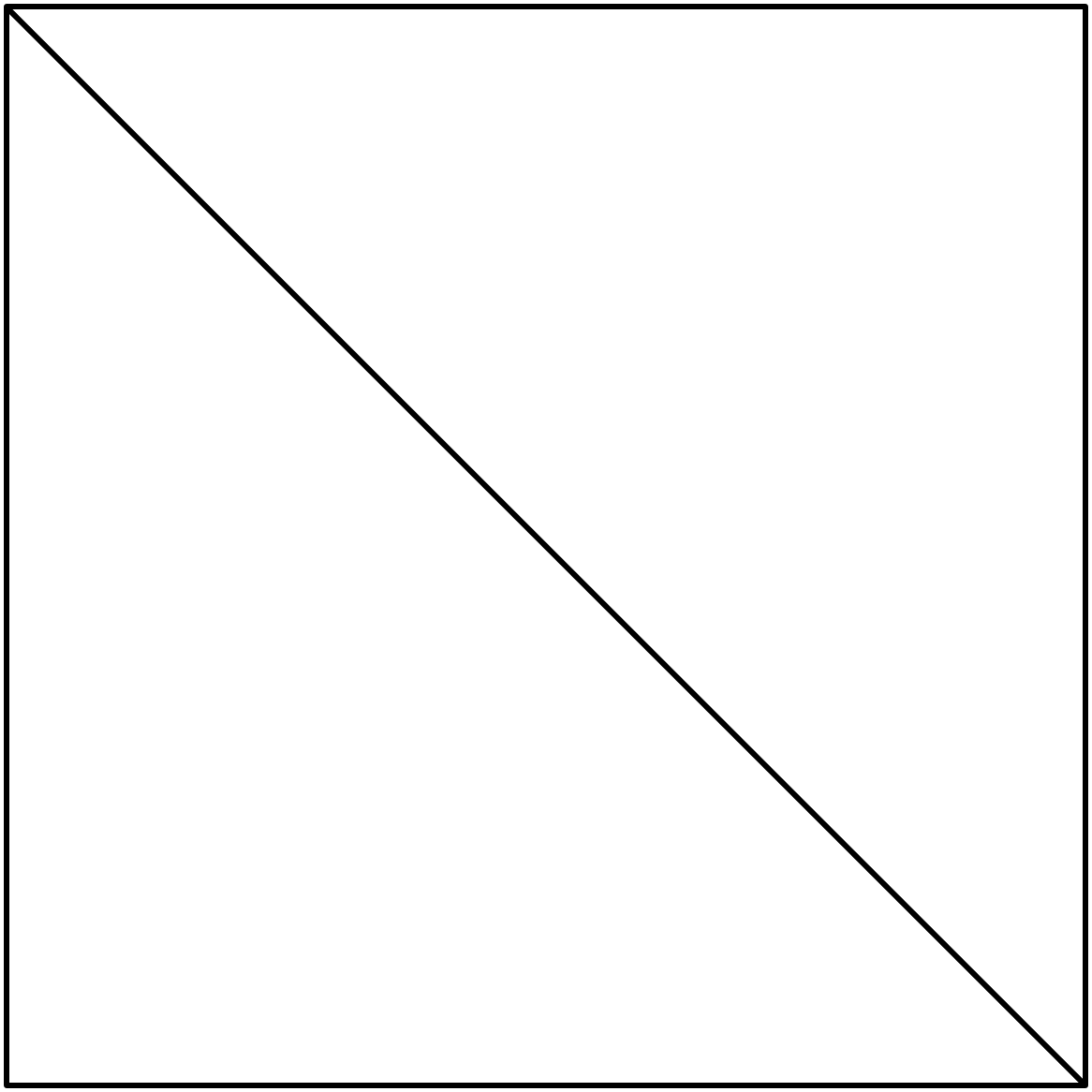
Have the students who completed the task by investigating how one-fourth pieces that are different shapes could represent one-fourth share what they did with the whole class. Encourage them to share examples.

Follow this explanation by the students with this for the whole class to consider - Today we investigated that we can divide a shape into half in different ways and end up with halves that are different shapes, like  and  . Is this ok? Why or why not?

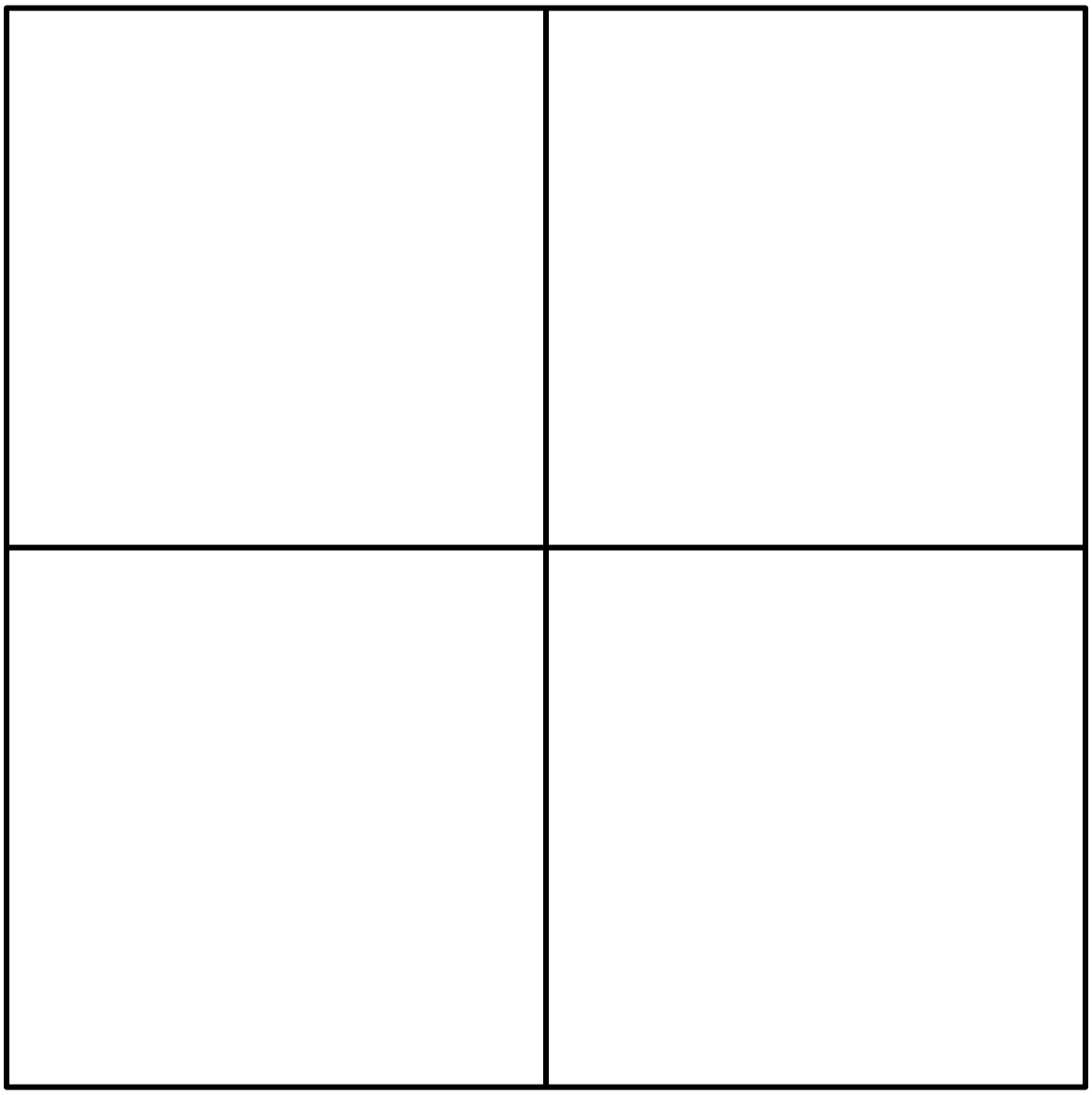
Sample student responses may include:

- Yes, it's ok because the two shapes cover one another (Ask students to show an example of this)
- Yes, it's ok because the squares you start with are the same and they are both broken in two equal pieces.





Shapes for *Extend the Task*



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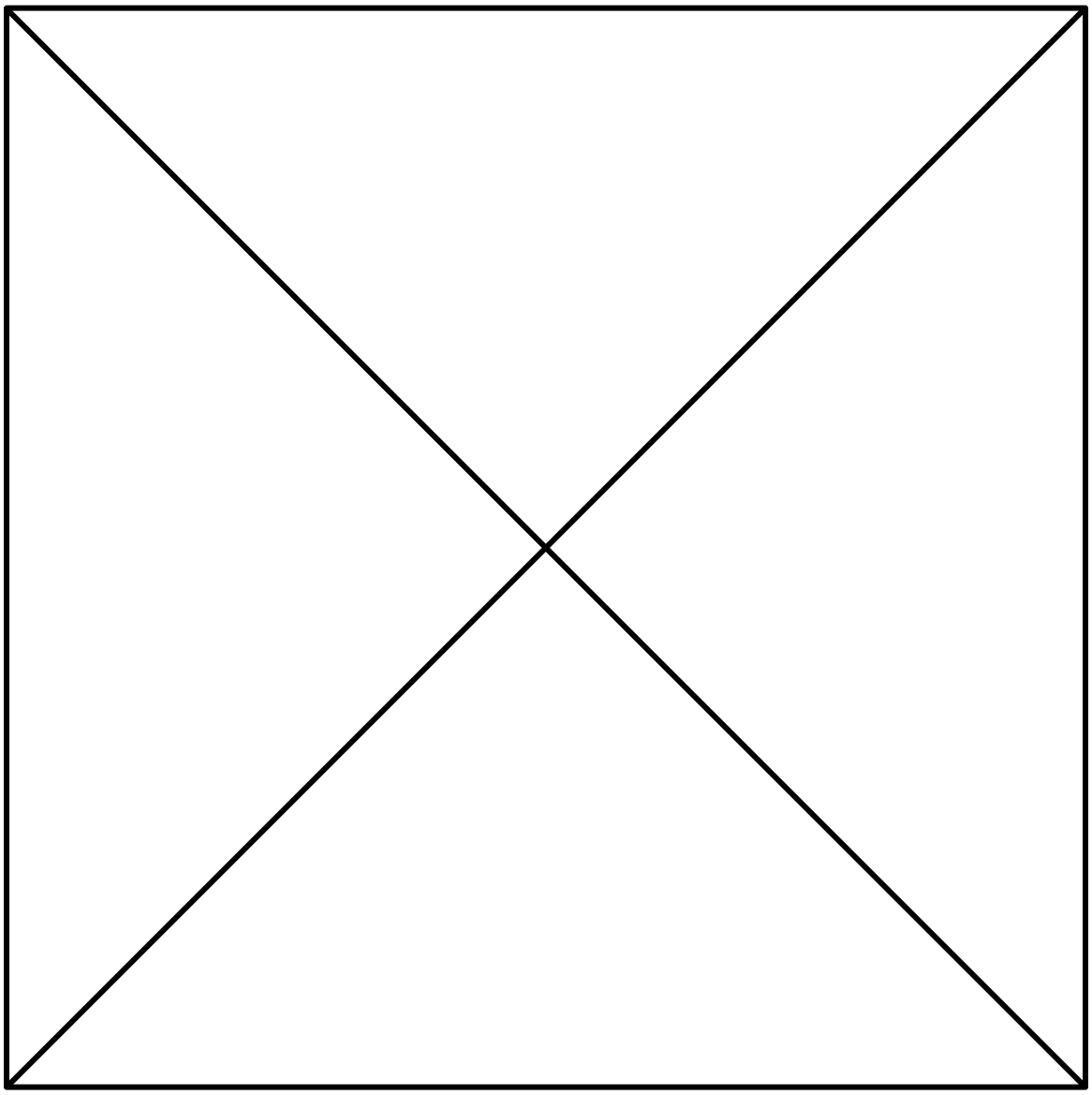

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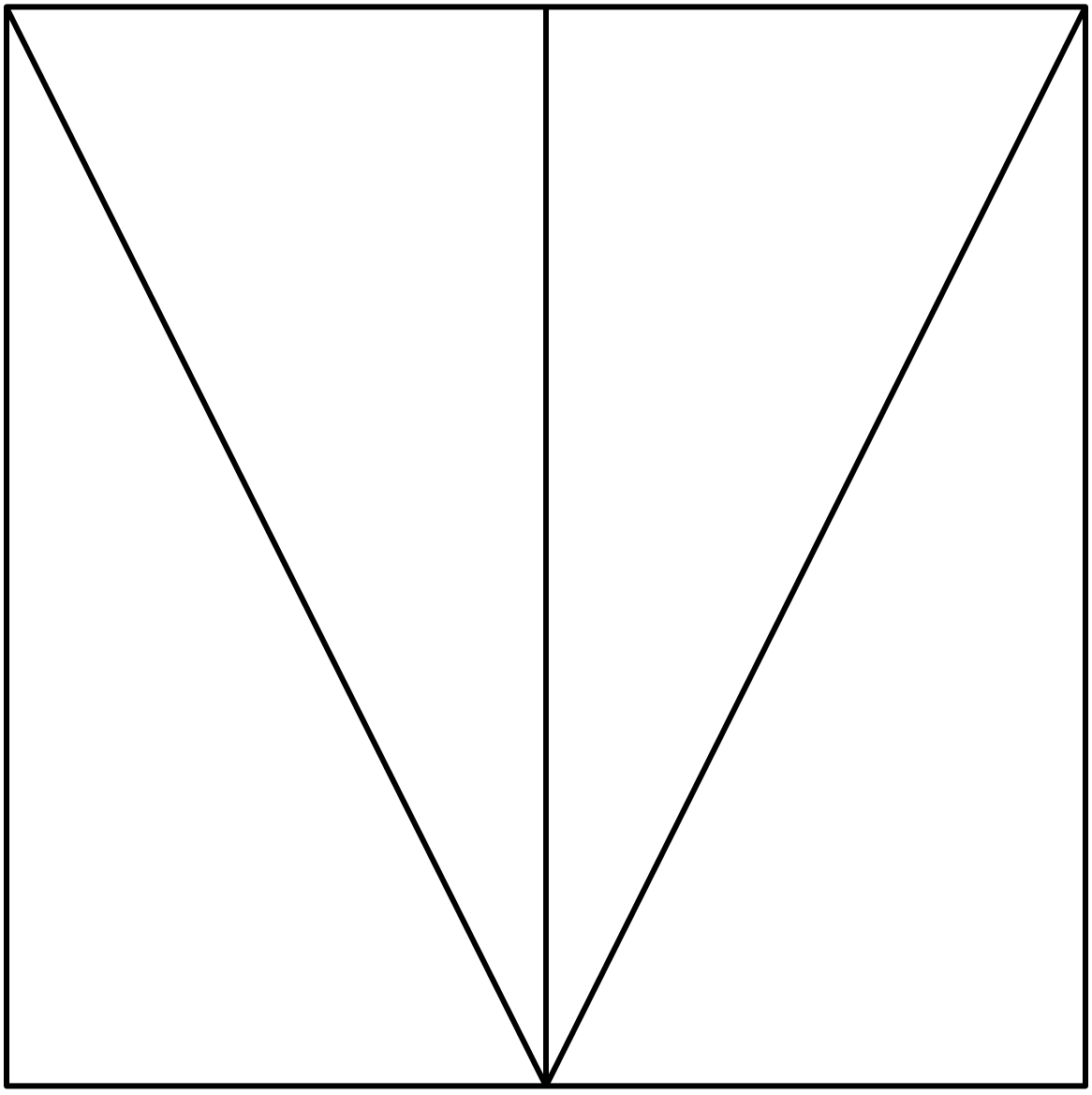

Shapes for *Extend the Task*