

## Lesson MK. 1

## How Many in Each Car?

In this lesson, students will use the F/F/O/E strategy to explore the decomposition of numbers to 10 . Students will use manipulatives to engage in small group and independent work to create different combinations of numbers to 10. They will represent their combinations through drawings, equations, numbers, pictures, or display of their manipulatives. They will be showing fluency by creating different combinations and will explain and elaborate on their combinations.

For more information about decomposing numbers, see chapter 1 (p. 15-20) in the Math Matters book.

## CCSS.MATH.CONTENT.K.OA.A. 3

Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5=2+3$ and $5=4+1$ ).

Standards for Mathematical Practice
MP2: Reason abstractly and quantitatively
MP4: Model with mathematics

Time Frame: ~60 minutes
To allow students to investigate the tasks and concepts in this lesson fully, it may take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered during the Explain section before moving to the Elaborate/Extend section.

## Materials

- Book - Bigmamas by Donald Crews
- Vehicle printouts on card stock, 2 colors (1 of each color per student)
- Wooden peg people (250 / class)
- Two-sided color counters (red/yellow) (10/student)
- Two-sided color counters - magnetic (1 set for the teacher)
- Two-Vehicle Recording Sheet - (2 copies per student, double-sided)
- Three-Vehicle Recording Extension Sheet - (1 copy per student, double-sided)


## Engage

Gather students to a class discussion space and ask, "How does your family travel or get places?" (If needed, say how do you get to the store or how do you travel to visit a relative's house.) Have several students share examples of the ways their family travels.

It is likely that at least one student would mention traveling by car, but if not be sure to bring this up after students have shared ideas. Prompt students to think about what it would be like for their whole family to travel by car. Questions to ask to guide this discussion include:

- How many people can fit in a car?
- Would everyone in your family fit in one car?
- What could you do if not everyone fit?

After several students share ideas as part of this discussion, read the Bigmamas book to the class. This is a story about a family who is traveling to visit their grandmother. As you read, point out the different modes of transportation. At the dining room table page, discuss the large family at the table and ask students how they all might travel somewhere together. This would be a good opportunity for students to share ideas with a partner before sharing out to the class. Encourage several students to share ideas during the whole class discussion. Highlighting when students share original ideas or many ideas may provide prompt student to use the Fluency, Flexibility, Originality, and Elaboration thinking skill.

## Olook For

- Students who express highly detailed explanations (communicative)
- Students who discuss original family stories and places of travel, or students who share many ways the family could travel (creative)
- Students who ask questions about other students' families or modes of transportation (curiosity)
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## Explore

To begin exploring number decomposition, students will work to decide how 5 people can ride in two cars. Provide each pair of students with 5 wooden peg people and two copies of the card stock vehicles (one of each color). Tell students "Using these 5 people and 2 vehicles, think about how they can travel in the two cars." Allow students several minutes to manipulate the people in different ways.

Walk around to listen to student conversations, and encourage students to solve the problem in multiple ways. Also, encourage students to discuss strategies and observations. Some possible questions to ask:

- What is another way the 5 people can ride in the cars?
- How many different ways can you find that the 5 people can ride in the cars?

OLook For?

- Students who develop solutions strategically (strategic)
- Students who model the problem multiple ways (creative)
- Students who clearly explain their thinking (communicative)


## Explain

Gather students to a class discussion space and facilitate a discussion of the ways that students decomposed 5 in the problem. Some questions to use to guide the discussion are as follows:

- What were some ways you arranged the people in two cars?
- How did you know how many people to put in each car? Encourage elaboration by asking students to "say more."

After students have shared all of the ways that they could decompose 5, say something like, "That is a lot to remember. Since you can't take the wooden people home, how might we show our work so you can share it with your families at home?" Have several students demonstrate different ways of recording their combinations on the Two-Vehicle Recording Sheet. This could be done on a document camera projector, Smartboard, or chart paper, or white board. Then, have all students go back to their seats to make one representation of a combination they had made with their wooden people on a recording sheet. Students can use any way of representing. Gather students back together to share the representations they created. Create an anchor chart of "Ways to Show our Math Thinking" by hanging a few different representations that students made (circles, stick figures, numbers, equations, etc.) on it.

## Olook For?

- Students who elaborate on their explanations of how many people they put in each car (communicative)
- Students who demonstrate an understanding of a pattern for the different combinations (perceptive)
- Students who represent their people and vehicles in original ways (creative)

Elaborate/Extend
Target Task
Gather students to a class discussion space and explain that more people have joined the family and need to travel together. Have students brainstorm who might have joined the family? (ex.: aunts, uncles, cousin, etc). Tell students that now there are 8 people in the family that have to travel together in the cars. Students should work with a partner to find as
many ways as possible that the family of 8 could travel in the two cars. Prompt students to keep track of the combinations on the Two-Vehicle Recording Sheet. Provide students with additional recording sheets if necessary.

## Extend the Task

For students who were observed engaging in one of the high-potential behaviors or who demonstrated advanced understanding of the concept in the Explore or Explain sections of the lesson, the target task can be extended by increasing the complexity of the task in one or more of these ways:

- Provide students with the Three-Vehicle Recording Sheet and ask students how the family of 8 could travel in 3 vehicles instead of 2 .
- Increase the number of people in the family that students will use to solve the problem (ex. a number greater than 10)
- Ask students to try to solve the problem without using the peg people manipulatives.


## Scaffold and Support

For students who may need some support in determining how to decompose 8 , some questions to scaffold students' thinking might include these:

- What are we trying to figure out in this problem?
- What materials could you use?
- How could you show 8 people in the cars?
- What is another way you could show 8 people in the cars?

OLook For:

- Students who use the anchor chart and other classroom materials to make their combinations (resourceful)
- Students who are able to make many combinations of a certain number to show fluency (perceptive)
- Students who notice the pattern of ascending/descending numbers in the two groups (perceptive and strategic)


## Evaluate

After students have had enough time to work on the task, bring them back to the meeting area to facilitate a whole-class discussion. Invite students to share multiple combinations that they discovered. As students share, the teacher can model the representations of how the students decomposed 8 for all students to see. Highlight student ideas that show originality and flexibility. Also, highlight when students share many ideas (fluency) or provide lots of detailed explanation (elaboration).

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## Lesson M1.1

## Counting Cupcakes

In this lesson students will be introduced to using the Visualization Thinking Strategy as an approach to solving word problems. Students are being asked to visualize and model the "action" in a story problem to recognize subtraction situations initially and later both addition and subtraction situations.

For more information about addition and subtraction problem contexts and solution strategies see chapter 3 ( $\mathrm{p} .55-67$ ) of the Math Matters book.

## CCSS.MATH.CONTENT.1.OA.A. 1

Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Standards for Mathematical Practice
MP1: Make sense of problems and persevere in solving them.

MP4: Model with mathematics.

Time Frame: ~ 60 minutes
To allow students to investigate the tasks and concepts in this lesson fully, it will likely take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered during the Explain section of the lesson before moving into the Elaborate/Extend section.

## Materials

A variety of math manipulatives (10 frames, counters, white board/marker, paper, etc.)
Pictures of piles of cupcakes
Subtraction action prompt cards (1 set per pair of students)
Addition action prompt cards (1 set per pair of students) Math journal or paper


Students are introduced to the visualization thinking strategy at this point in the lesson using a non-math example and then explore how it could be used in mathematics.

## Engage

Ask students to think of a time when they had to picture or imagine something in their mind. Provide an example, if needed, such as "Sometimes people ask me to tell them about my dog, Bruce. When I tell them what he looks like, it helps if I picture him in my head. Bruce is a big dog with a black spot on his eye." Ask a few students to share some examples of things that they have been able to picture.

Tell students that today, we will practice visualizing or making pictures in our minds to help us with math. Prompt students to picture a giant pile of cupcakes in the center of the room. The teacher could say, for example, "Picture the largest pile of cupcakes you have ever seen. Once you can picture it...What do you see? What details are you picturing? What colors do you see? How big is the pile?" Have students share what they are picturing with a partner, and then ask multiple students to share some of their visualizations with the class.
[Use one of the sample pictures of a pile of cupcakes if needed to help prompt students' thinking, but do not START with the existing pictures. Be sure to encourage creative exploration of different ideas of what the image might look like.]

Ask students to imagine what might happen if the teacher in the room full of cupcakes left the room. Prompt students to picture this event in their mind a giant pile of cupcakes in the classroom, students in the classroom, and the teacher leaving the classroom. Ask students if the pile of cupcakes is likely to get bigger or smaller while the teacher is gone. Tell the students that when the teacher comes back, the cupcake pile is smaller. Ask students to imagine what might have happened to some of the cupcakes. Have students share their ideas about what happened to the cupcakes with a partner and then have student pairs share a few of their ideas with the whole group. Write the actions the students describe on a chart or whiteboard (e.g., eaten, stolen, sold, smushed...)

Explain that today we will use the strategy of visualization to help comprehend the actions in a word problem.

## OClook For

- Students who share a wide variety of ways or unique ways the pile of cupcakes changed, or those with unusual detail in descriptions of the cupcakes (creative)
- Students who express highly detailed explanations (communicative)


Students use the visualization thinking strategy to picture subtraction situations using cupcakes.

## Explore

Students will continue to use visualization as related to subtraction in this section. Invite students to imagine that they have 10 cupcakes with their partner. Show students one of the action prompt cards that requires subtraction (see end of lesson) and ask them to describe what they see when they picture their cupcakes and this action happening.

Tell students that they are going to do this multiple times with their partner. Provide each pair with a set of subtraction action cards. Tell students that they should picture their 10 cupcakes and then picture the action occurring. They should then describe to their partner what they see and what happens to the pile of cupcakes. For example, if the card says someone ate 6 cupcakes the student should describe picturing someone eating 6 of the 10 cupcakes and that the pile has fewer cupcakes as a result (some students might determine how many cupcakes are left). Students should continue, each time picturing the 10 cupcakes again and then what would happen with the action on the card.

Observe as the pairs of students use the action prompts and describe what is happening to their imagined piles of cupcakes. Students should also be prompted to model what is happening to their cupcakes by drawing, acting out, or modeling with manipulatives.

## OClook For

- Students who model the problem in multiple ways (draw, act out, write). (Creative)
- Students who can clearly explain the connection between their visualization and the operation. (Communicative)


## Explain

As pairs of students complete the task, bring the class back together and facilitate the sharing of students' models and explanations. Some questions that could be used to guide the discussion are as follows:

- How did the image that you pictured change after the action took place?
- How did picturing the action in your mind help you to solve the problem?
- What is similar between all of these actions or their results?
- What did you show in your model?
- How might we show this with numbers and symbols?

Encourage multiple students/pairs to share ideas. Students could also be asked to look for similarities/differences in what is happening in their situation and the result for their pile of cupcakes.

## Olook For

- Students who are able to clearly explain their thinking related to how they completed the task. (Communicative)
- Students who are able to clearly explain their thinking related to how visualization could be used for solving a problem. (Communicative)
- Students who focus on the most relevant details for solving a problem as they explain how they visualize. (Strategic)


## Elaborate/Extend

Target Task
Have students repeat the task from the explore part of the lesson, but this time they should picture a pile of 20 cupcakes. Also provide students with both the addition and subtraction action cards.

Observe as the pairs of students use the action prompts and describe what is happening to their imagined piles of cupcakes, again returning to a starting point of picturing 20 cupcakes each time. Students should also be prompted to model what is happening to their cupcakes by drawing, acting out, or modeling with manipulatives.

## Extend the Task

For students who were observed engaging in one of the high-potential behaviors or demonstrated advanced understanding of the concept in the Explore or Explain sections of the lesson, this target task can be extended by having students try one of these tasks:

- Turn over two (or more) cards, then picture the sequence of events, and determine how many cupcakes would be visible after both of the actions. (For example, starting from 20, students would explore what happens when they take away 5 and then add 6 and then any further steps in sequence.)
- Generate additional addition or subtraction action cards that could be added to the pile.

Scaffold and Support
For students who may need some support in determining how many cupcakes are visible after the action occurs, encourage them to use manipulatives to model the action. Also, consider modeling with one of the sample pictures of 6,8 , or 12 cupcakes for practice and then encourage
students to work with the visualization of 20 cupcakes.

## Look For

- Students who represent or solve the problem in multiple ways. (Creative)
- Students who work through the problem strategically. (Strategic)


## Evaluate

As students complete the task, bring the class back together and facilitate a discussion about how they visualized the situations. Encourage each partner group to share at least one example. As the pairs are sharing the teacher should be sure to ask if they added to or subtracted from their original pile of cupcakes with the actions they did. Create a chart on the board/chart paper with the students' ideas about which actions would be addition and which would be subtraction. Students should discuss what is similar on the addition and subtraction chart and be able to name the actions as either subtraction or addition.


|  | Sarah received 8 more cupcakes. |
| :---: | :---: |
| $\begin{gathered} \text { Mia brought } 5 \text { more } \\ \text { cupcakes. } \end{gathered}$ | $\dot{0}$ <br> y <br> 0 <br> 0 <br> 0 <br> 0 <br> 0 <br> 0 <br> 0 <br> 0 <br> 4 <br> 0 <br> 0 |


|  |  |
| :---: | :---: |
|  | Billy dropped 2 cupcakes. |








## Lesson M1.2 Balancing Act

In this lesson, students will explore the meaning of the equal symbol used in equations. A common misconception about the equal symbol is that it indicates that the answer comes next. Instead, the equal symbol represents the idea that the expressions on each side of the equation have the same value. Students engage in developing this understanding by using a balance scale/pan balance because this can demonstrate that the value on each side of the equal sign is the same. Students also will engage in the Encapsulation thinking strategy during this lesson as they are asked to explain their understanding of the equal symbol.

For further discussion of the concept of the equal symbol please see the discussion of "equality" on page 195 of the Math Matters book.

CCSS.MATH.CONTENT.1.OA.D.7: Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.

Standards for Mathematical Practice
MP7: Look for and make use of structure

Time Frame: ~ 60 minutes
To allow students to investigate the tasks and concepts in this lesson fully, it will likely take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered during the Explain section of the lesson before moving into the Elaborate/Extend section.

## Materials

1 pan balance for each pair/small group
Assorted colors of one inch cubes for each group (20 cubes for each group)
Chart paper, markers
Dice (at least 2 per pair)
Number images on cards
Copies of balance scale diagram

## $F$

This part of the lesson prepares students to engage in encapsulation because they are talking here about the meaning of equality (though not yet with specific reference to the equal sign).

## Engage

Begin the lesson by engaging students in understanding that numbers can be represented in different ways. Pass out one number image card to each student and ask students to find at least one other person whose card shows the same value (ex. the digit 5 and 5 popsicle sticks, or 2 Lego blocks and 2 counting bears). Students may find multiple matches depending on the cards that are used. When setting up the lesson be sure to plan that each student would have at least one match.

Bring the class back together and have the students share out how the images in their pictures match. Note that you may have some students who matched cards based on the material (ex. Popsicle sticks, blocks, etc.). These matches are reasonable to students since they typically sort objects in this way, but encourage students to focus on the representations of the quantities.

## Explore

Introduce students to the balance scale and have students share ideas for what it could be used for. Consider having students partner talk and then share out their ideas to the group.

Provide time for groups (or pairs) of students to explore the use of the balance scale. Have each group gather the materials needed (1 balance scale, set of 1-inch cubes in different colors).

After several minutes of open exploration, pose this question to students "Using the blocks, how do you think we could balance the scale?" Allow students several minutes to explore how to make the scale balance. Students might group the blocks by color and put the same number in different colors on each side of the balance scale. For example, students might put 8 red blocks on one side and 8 blue blocks on the other side. Alternately, students might mix the colors of the manipulatives on one or both sides, such as 8 red blocks on one side and 5 blue and 3 yellow blocks on the other side, or 4 red blocks and 4 green blocks on one side and 7 blue blocks and 1 yellow block on the other side.

As students are working, circulate, ask questions, and listen to student discourse. Note when students use words like balance or looking for equal amounts, etc. Students might bring up the idea of equal weight; however, because students are using the same manipulatives on each side of the balance scale, encourage them to think about the quantity on each side.

## OClook For

As students are working to balance their scale, take note of

- Students that are able to balance the scale in multiple ways or show ways that are different from other groups. (Creative)
- Students who are able to elaborate on explanations for how the scale is balanced. (Communicative)


## Explain

After each group (or pair) has developed at least one way to balance the scale, have them share out one way that they made the scale balance and explain why the scale is balanced.

Then, tell students you noticed that they came up with many ways to show that the scale can be balanced, but you are wondering if there is a way that we could document or record how their scale is balanced. Show an example balance scale from the groups' ideas, such as 8 red blocks on one side and 5 blue and 3 yellow blocks on the other side, and ask students to talk to a partner about how they could document or record what they see on the scale.

As students make suggestions, draw or write them on a chart or white board. These are some ideas students might come up with:

- Draw a picture

- Use numbers


Ask students how this balance can be represented with a symbol or what could be added to the examples that would help us show that both sides of the balance have the same quantity or value. Allow the term "equal sign" to come from the students. If students do not come up with it, draw it in to one of the examples and ask them what it means.

Ask groups (or pairs) to come up with examples of how they could document or record what their balance scale looked like using a diagram of

## $F$

Students are beginning to engage in encapsulation at this point of the lesson because they are looking at multiple examples of when the equal sign can be used.

the balance scale like those shown above. Students can develop more than one way to represent what their balance scale looked like. Encourage students to explore what happens when they switch the sides where the blocks are. Ask them if it changes the quantity and if it changes the way they would show it on paper.

Have a few groups share the ways that they represented their balance scale.
Encourage students to be sure they used the equal sign in their representation. Also explain that mathematicians use the word equal to describe what today we have described as sides "are the same," or "have the same value."

## OOLook for

- Students who can recognize patterns in the ways that numbers can be represented. (Perceptive)
- Students who start explaining the communicative property of addition (although they likely will not know its name) based on what they notice about moving the manipulatives/numbers from one side of the scale to the other. (Resourceful)


## Elaborate/Extend

## Target Task

Introduce the following activity to the students. Roll two dice, and record the resulting roll on the balance scale diagram as the number of dots on one die plus the number of dots on the other die. For example: represent $\because::$ as $2+4$ on one side of the balance scale diagram. Some students may still need to represent these using manipulatives at this point; allow them to do that and also record the numbers on the diagram. Ask students to create another expression, or way to show six, on the other side of the balance scale. Encourage students not just to use the number six, but to find other ways to make six such as $1+5$.

Provide pairs of students with two dice and copies of the balance scale diagram. Students will roll the two dice, determine the sum of the two numbers rolled, and represent the sum in multiple ways on the record sheet. These may be recorded using pictures and/or written numerals. Students may need multiple pages if they come up with multiple representations.

Extend the Task
For students who were observed engaging in one of the high-potential
behaviors or demonstrated advanced understanding of the concept in the Explore or Explain sections of the lesson, this target task can be extended by having students:

- Encourage students to represent both sides of the equation using an addition expression instead of the sum (for example, $3+4=5+2$ ).
- Encourage students to roll more than two dice to determine the initial sum. This engages them in working with more than two addends.
- Have students find as many ways as possible to express an equality to a set value.


## Scaffold and Support

Have students complete a missing addend equation to represent what was rolled (e.g., If I have $8=3+\ldots$ ) and explain how a change to one side affects both sides (if I have $8=3+5$, what happens when I now have $7=$ __?)

## OClook For

- Students that are able to balance the scale in multiple ways or show ways that are different from other groups. (Creative)
- Students who begin to represent expressions using more than two addends (ex. $3+4=2+1+4$ ). (Perceptive)
- Students who begin to represent expressions using different operations (ex. $2+5=8-1$ ). (Curious)


## Evaluate

As students complete the task, bring the class back together and facilitate a discussion about their understanding of the meaning of the equal sign as a symbol. Students have had multiple experiences in this lesson that could lead them to understand that the equal symbol represents that there is the same value or quantity on each side of an equation.

Pose the question, "What does the equal sign mean?" to the students and encourage students to share ideas with a partner. Have each pair share out their ideas and an example of their representations to help them explain what the equal symbol means. Record student ideas on the white board or chart paper.

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## Number images for cards





## Lesson M1.3 Rolling with Place Value

In this lesson, students will explore the value of the digits in a two-digit number and develop an understanding that each digit represents the numbers of tens and ones in a number. Additionally, students will have the opportunity to use mental math to determine 10 more than a two-digit number. The decisions and outcomes thinking strategy will be used as a way for students to analyze a game-based situation in which they try to reach a target number strategically, using place value and making sure not to go over the number. On each turn of the game, students will have to make a decision about how to get to the target number and then they will evaluate the outcome of each choice.

For information about place value in our base-ten number system see the Math Matters book (p. 25-30).

## CCSS.MATH.CONTENT.1.NBT.B. 2

Understand that the two digits of a two-digit number represent amounts of tens and ones.

## CCSS.MATH.CONTENT.1.NBT.C. 5

Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

## Standards for Mathematical Practice

MP1: Make sense of problems and persevere in solving them. MP5: Use appropriate tools strategically.

Time Frame: ~ 60 minutes
To allow students to investigate the concepts in this lesson fully, it may take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered during the Explain section of the lesson before moving into the Elaborate/Extend section.

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Materials
0-9 Dice (1 per student)
10-90 multiples of 10 dice (1 per student)
100-sided die (1 per class)
Base Ten Place Value Frame (5 per class)
Base Ten Blocks
120 Chart (1 per student)
200s Chart (for the Extend the Task)
Plastic Sheet Protectors (1 per student)
Dry Erase markers (1 per pair)
Color marker chips (1 chip per student)
Digital Copy of Sample Game Board (end of lesson)
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## Engage

Gather students in a space in the classroom that is conducive to discussions and begin the lesson by writing the number 37 in large print in a place where all students can see it (white board, chart paper, Interactive White Board [IWB]), but do not tell them what the number is. Ask students, "What do you know about this number?"

Give students time to share their ideas with a partner and listen in as the pairs talk. Ask students to share their ideas with the class and document these ideas for all students to see. Students might come up with some of the following ideas:

- The number is "thirty-seven"
- The number 36 comes before it
- The number 38 comes after it
- There are 3 tens in the number
- There are 7 ones in the number
- The number is bigger than 30

If students do not express the ideas related to place values (the number of tens and ones), ask questions that will lead students to think about these ideas. After this discussion, tell students that they are going to work in pairs to decide how they will represent numbers using base-ten blocks.

## OOLook For

- Students who are able to explain that the number can be decomposed into tens and ones. (strategic)
- Students who share multiple mathematically valid ideas. (creative)


## Explore

For this part of the lesson students will work in pairs and will need the following materials:

- One 0-9 ones die
- One 10-90 multiples of ten die
- Base-ten blocks (tens and ones)

Give each pair one of each type of dice. Ask students what they notice about the dice and have several students share their observations. Tell students that they will roll both of their dice to come up with a 2-digit number, and then decide how they will represent the number using base-10 blocks. At this point of the lesson, students may represent their numbers in different ways. For example, some students may use the correct number of tens and ones, while other students may use all ones. This will be discussed in the Explain part of the lesson.

Set a timer for 5 minutes for exploration. During this time, students may roll multiple times to create different numbers.

## Onlook For

- Students who represent the numbers in multiple ways. (creative)
- Students who are using the regrouping strategy (strategic)
- Students who are collaborating well as a team (communicative)


## Explain

Bring students back together and ask: "How did you decide to make the numbers you rolled by using the base ten blocks?" Have students discuss with the class. Be sure to focus the discussion on the decisions students made in relation to the concepts of place value. For example, a student might say that to make the number 83 they decided to use 8 tens because the digit 8 is in the tens place. Also emphasize the use of precise language "the digit 8 " was in the tens place, not "the number 8 ."

Document the decision ideas that students discuss in a place that all students can see them.

## Olook For

- Students who are communicating their process for representing the numbers they rolled. (communicative)
- Students who develop and share multiple mathematically valid ideas. (creative)


## Elaborate/Extend

Tell students that they are going to play a new game in which the goal is to reach a target number on the 120 chart. Show students the 120 chart and ask what they notice about it before explaining the game to them.

## Target Task

Students should play this game in pairs. Give each pair the following materials:

- A 120 chart for each student in a plastic sheet
- A dry erase marker for the pair
- One 0-9 die
- One 10-90 multiple of 10 die

Explain the game to the students and then model one round of the game.

- The goal of the game is to roll the dice to get to the target number without going over it. The mathematical and strategic goal is for students to consider the place value of their target number and the values of the numbers on the dice to make decisions about if they should roll the 0-9 die or the 10-90 die to reach a target number without going over it.
- In the pair, one student will roll the 0-9 die and the other student will roll the 10-90 die. Students will combine their rolls to determine the target two-digit number. (For example, one student rolls 40 and the other student rolls 3 . The target number for that round would be 43.) Students should mark this target number on both of their boards by circling it.
- Each student will start by placing a color chip on the 0 of their own board. At the start of each turn, the student should color in the box where the chip is (so they know where it started on that turn). Next the student will decide if they want to roll the 0-9 die or the 10-90 die. They can only roll one die on each turn. After rolling, the student will count on that many spaces and place the chip on the new number. If the student goes over the target number, they must return to the marked space from the start of that turn and their turn ends.
- Encourage students to make decisions about which die they should roll and how that might affect the outcome on each turn. For example: If the target number is 72 and the student is at 50 , should they roll the 0-9 die or the 10-90 die to get close to the number without going over? Why?

Set a timer for 10 minutes for the students to play the game. When students complete a round of the game, they should start a new round by setting a new target number. They should have the opportunity to play multiple rounds within the time frame.

Observe as the pairs of students are making decisions about which die to roll. Students should also be prompted to share their thinking about their decision before each roll and the outcome of that decision (e.g., did the
student go over the target number?, what helped you decide which die to use on each roll?, etc.)

## Olook For

- Students who are making strategic decisions before they roll their die. (strategic)
- Students who are able to come close to their target number without going over by accurately weighing the outcomes. (strategic)

Extend the Task
For students who were observed engaging in one of the high-potential behaviors or who demonstrated advanced understanding of the related place value concepts, the target task can be extended by having students use the 200s chart as the game board instead of the 120 chart. Play would proceed as follows:

- Have students roll the 100-sided die, the 10-90 die, and the 0-9 die once and add the results together to determine the target number.
- Students would play the game as in the target task to decide if they want to roll the 10-90 or 0-9 dice on each turn, and they also can decide if they want to roll that same die once or twice on the same turn, but they cannot change which die they roll on a turn. However, if they go over the target number on the second roll they have to go back to the number they started on at the beginning of the turn and their turn ends.


## Scaffold and Support

For students who may need some support with place value provide them a base ten place value frame and 10 tens sticks and 10 ones blocks. The students will be able to use the frame to place their tens and ones and then represent their number using the tiles below.

## Evaluate

After students play multiple rounds of the game, bring the class back together and facilitate a discussion about how they made their decisions about which dice to roll on each turn and the different outcomes that their decisions led to. Encourage each partner group to share at least one example. On a piece of chart paper or white board the teacher records examples of the different decisions that the students share.

Project the Sample Game Board for all students to see. Tell the students that this board is one from another class and their job is to use the markings on to board to figure out what decisions were made by the player for each turn based on the outcome. Point to the 0 and tell students that this is where they started. Then point to the circled number, 58 and ask what this number
was. Based on having to circle the target number when they played their games, students should identify 58 as the target number. Also tell students that this student playing this game noted which roll for each turn with a small circled number. For example, the circled 4 on the number 45 means that was that student's fourth turn.

Ask students to find the result of the first roll. Students should identify the 70 because it has a circled 1 on it. Ask students what decision they think was made about which die to roll. Students should conclude that the 10-90 die was rolled. Tell students you notice that it is crossed off and ask why that might be. They should conclude that it is greater than 58 so it is above the target number and the student would have to go back to 0 . Do this for each of the numbered spaces on the chart to have students analyze the decisions that were made based on the recorded outcomes for this sample game.

Return to the list of strategy decisions the students developed and ask students how these strategies helped them make decisions. Be sure to ask students to make connections between their strategies and knowing about the number of 10 s and ones in the numbers and/or on the dice.

120 Chart

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |
| 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 |
| 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 |
| 110 | 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 |
| 120 | 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 |

## 200 Chart

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |
| 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 |
| 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 |
| 110 | 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 |
| 120 | 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 |
| 130 | 131 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 |
| 140 | 141 | 142 | 143 | 144 | 145 | 146 | 147 | 148 | 149 |
| 150 | 151 | 152 | 153 | 154 | 1155 | 156 | 157 | 158 | 159 |
| 160 | 161 | 162 | 163 | 164 | 165 | 166 | 167 | 168 | 169 |
| 170 | 171 | 172 | 173 | 174 | 175 | 176 | 177 | 178 | 179 |
| 180 | 181 | 182 | 183 | 184 | 185 | 186 | 187 | 188 | 189 |
| 190 | 191 | 192 | 193 | 194 | 195 | 196 | 197 | 198 | 199 |
| 200 | 201 | 202 | 203 | 204 | 205 | 206 | 207 | 208 | 209 |

120 Chart

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| $2_{40}$ | $3_{41}$ | 42 | 43 | 44 | $44^{4}$ | 46 | 47 | 48 | 49 |
| 50 | 51 | 5 | 52 | 53 | 54 | 55 | 56 | 57 | 58 |
| 60 | 7 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 40 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |
| 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 |
| 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 |
| 110 | 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 |
| 120 | 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 |



## Lesson M1.4

## How Efficient Are You?

In this lesson, students will explore different ways to determine the value of expressions with efficiency. Students may determine the sum using their choice of strategies and mathematical tools (e.g., number lines, ten frames, manipulatives). Students will engage in the Point of View thinking strategy when they share and defend their own mathematical ideas and evaluate the mathematical ideas of others.

Please see pages 60-67 of the Math Matters book for more information about modeling, counting, and number sense based strategies for addition.

## CCSS.MATH.CONTENT.1.NBT.C. 4

Add within 100, including adding a two-digit number and a onedigit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding twodigit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

Standards for Mathematical Practice
MP3: Construct viable arguments and critique the reasoning of others.
MP5: Use appropriate tools strategically.

Time Frame ~ 60-75 minutes
If the lesson will stretch over two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered during the Explain section of the lesson before moving into the Elaborate/Extend section.

## Materials

Dry erase plastic sleeves/ Dry erase markers
Reproducible sheet that is split into four sections (Ten frame board, number line (0-20), number bond, and blank space for work)
Most Efficient Path Map
Digital copy of Most Efficient Path Map
Red and yellow sided counter chips
Elaborate/Extend Record Sheet and Extend the Task Record Sheet Unnumbered Number lines with tick marks

## Engage

Students will begin at their desks and look at the Most Efficient Path map. Have the students draw a path from the school to the park (do not give any further direction - just a path from school to the park). After students are done drawing, ask several students to show and explain the path they drew. You could have students draw the path on the map that is being displayed for the class in different colors. If there is not much variety in student paths, encourage them to think about other possible paths.

Ask students to share what they notice about the paths and have them share their ideas. Some ideas students may share include:

- Some paths are really windy
- Some paths are really long
- One path is really short

After all ideas are shared, if students did not come up with this idea, tell students you noticed something about one of the paths, that if they took that path it would probably take the least amount of time to walk or bike to the park. Ask students to identify this path and then compare their path to the shortest path. Ask them if they would change their path, and why or why not. Have students share out their point of view by explaining their reasoning behind which path they would choose.

Introduce the word efficient to the students. This can be done by writing it on the board/chart paper or displaying a word wall card with the word efficient on it. Ask students if they have heard the word efficient before. Have multiple students share what they think efficient means and/or have students come up with examples of being efficient that would be relevant to them (ex. getting up for all the materials they need for a task at once instead of having to get up multiple times for the materials). Since efficiency is a new concept for most students at this time, they may need help thinking of examples.

## Olook For

- Students who justify efficiency in unique ways (for example, I have a bike so the flat path is most efficient). (creative)
- Students who use the provided scenarios to generate a definition of efficient (for example, this path would take the least number of steps so efficient probably means doing something quicker, with fewer steps). (resourceful)


## Explore

Tell the students that the class is going to work on a math situation to help us understand more about what the word efficient means. Gather and display the materials/manipulatives that students can use to show how
they can add or group objects to solve the problem. Tell students that they can use the materials of their choice to do their work during the lesson.

Project or write the expression 9+6 on the board/chart paper. Ask students to show how they might use these materials to solve the expression $9+6$. Give students time (approximately 5-8 minutes) to use the materials (strategy sheet) to determine the value of the expression $9+6$ independently. Circulate to observe the materials and methods students use.

## Olook For

- Students who find efficient strategies like making a ten. (Strategic)
- Students who identify and describe patterns and connections. (Perceptive)
- Students who reflect on a strategy that had previously tried and make modifications based on efficiency. (Resourceful)


## Explain

When students have completed the Explore task, ask for volunteers to share out the value of the expression they came up with. Write all responses (correct and incorrect) on the board or chart paper. Tell students they will then decide as a class which is the accurate value by talking about the strategies they used.

Have students first turn and talk to the person next to them to share how they showed their work. While students are talking, listen for different strategies students have used to determine the value of $9+6$. Ask several students to share their strategies and then ask others to repeat/summarize the different strategies they heard. Have the class come to a conclusion about what the correct value of $9+6$ is.

Explain that mathematicians like to work efficiently, or working in a wellorganized and smart way with available tools but without a lot of extra steps. Return students' attention to the strategies they used to add or group objects together and ask them to think about efficiency. The questions could be phrased as "For who would $\qquad$ strategy be more efficient for than $\qquad$ strategy? Why do you think so?" or "Someone who knows all their make ten facts would find what strategy most efficient? Why do you think so?" Invite students to add on to ideas that come up or to agree or disagree with the ideas presented and explain why they agree or disagree. Note that at this point students might all think the strategy they used is the most efficient because it is the one they are comfortable with. As long as they can justify efficiency based on the tools or steps/skills they possess is a good first understanding of efficiency.

## OO Look For

- Students who apply their understanding of similarities and differences between the different strategies. (Perceptive)
- Students who clearly articulate an idea. (Communicative)
- Students who investigate alternative solutions to solve the problem. (Strategic)
- Students who identify and describe patterns and connections. (Perceptive)

Note - Determining the value of this expression engages students in the standard highlighted for this lesson and adding a two-digit number to a multiple of 10 .

## Elaborate/Extend

Target Task
Project or write the expression 19+7 on the board/chart paper. Tell students they will be finding the value of 19+7 and can use the strategies/materials they want, but they need to show their work in two different ways/with different materials. Give students time to work independently or with a partner on this task (approximately 10-12 minutes) and have them document their work on the Elaborate/Extend worksheet under Strategy 1 and Strategy 2. As students are working, observe the strategies being used, ask students if they think the strategies are efficient, and check that they came up with the correct value for 19+7.

## Extend the Task

For students who are observed engaging in one of the high-potential behaviors or demonstrated advanced understanding of the concept in the Explore or Explain sections, this target task can be extended by increasing the complexity of the task. Instead of the expression in the target task, present students with the expression 19+7+20 and have students determine the value of this expression. Tell students they can use the strategies/materials they want, but they need to show their work in two different ways/with different materials. Give students time to work independently or with a partner on this task (approximately 10-12 minutes) and have them document their work on the Extend the Task worksheet under Strategy 1 and Strategy 2. As students are working, observe the strategies being used, ask students if they think the strategies are efficient, and check that they came up with the correct value for $19+7+20$.

## Scaffolding and Support

For students who may need some support determining the value of 19+7, help students decide which strategies to use. Be careful not to tell students which strategy to use, instead use questions to help students make their own decision about the strategy they are comfortable using. If students need further scaffolding have them practice 9+7 instead of $19+7$ and see if they begin to recognize a pattern between 9+7 and 9+6.


Students are engaged in the Point of View thinking strategy during this discussion because they are starting to convince others based on what they believe is the most efficient strategy.

## OOLook For

- Students who adapt strategies in a new and different way and transfer understanding to new situation (Resourceful)
- Students who investigate alternative strategies to solve the problem. (Strategic)
- Students who identify and describe patterns and connections. (Perceptive)


## Evaluate

When students have completed the Elaborate/Extend task, remind students about what efficient means. Have students review the two strategies they recorded on their record sheet and ask them to explain what type of math student each strategy would be most efficient for. Have the students turn and talk about how the strategy they circled is the most efficient for a particular problem from their point of view. Have multiple students share their thinking with the class about how their strategy is efficient for them and explain why. Students should be prompted to agree or disagree and provide their reason(s) why.

Elaborate/Extend Record Sheet
Name:

| What is the value of 19+7? |  |
| :---: | :---: |
| Strategy 1 |  |

Extend the Task Record Sheet
Name:

What is the value of $19+7+20$ ?

| Strategy 1 | Strategy 2 |
| :--- | :--- |

Circle: Which strategy is the most efficient?




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## Lesson M1.5 <br> Make a Half and a Fourth

In this lesson, students will explore ways to partition rectangles and circles into two and four equal shares. Students may complete the tasks in different ways, and individual students may use multiple methods. The Encapsulation Thinking Strategy is exemplified in the discussion parts of the lesson, especially in the Evaluate section. Students should precisely explain their reasoning related to the idea that halves have to be two equal shares of the whole and fourths need to be four equal shares of the whole.

For more information related to fractions as part of a whole, see the Math Matters book (p. 99-105 \& 114-119).

## CCSS.MATH.CONTENT.1.G.A. 3

Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Standards for Mathematical Practice
MP3. Construct viable arguments and critique the reasoning of others.

MP5. Use appropriate tools strategically.

Time Frame: ~ 60 minutes
To allow students to investigate the tasks and concepts in this lesson fully, it will likely take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered during the Explain section of the lesson before moving into the Elaborate/Extend section.

## Materials

Sandwich (or picture of one)
Copies of shapes included in this lesson (cut out prior to lesson)

```
Square tiles
Scissors
Chart paper/markers
Tape
```


## Engage

Ask students to tell about a time when they had to share something. Invite them to partner talk and then share out some of their examples.

Follow this discussion by asking students what these sharing situations have in common. Record student ideas on a chart paper or white board. The goal here is for the students to come up with the idea that when we are sharing things fairly, each person gets the same amount or size of what is being shared.

## Explore

Show students a sandwich (actual or image below). Pose a situation such as the following to them:

I brought in a sandwich today and wanted to share it with one of my friends at lunch. I want to be sure that we are sharing it fairly so that we both have the same amount of the sandwich.

Ask students how we might be able to share the sandwich fairly. Allow for some think time and then ask students to share with a partner. Listen in to ideas that the students are sharing with one another and note ideas that should be shared with the whole class.

Have students share out their ideas. They may demonstrate using a paper "sandwich."

Provide pairs of students with different shapes (squares, rectangles, and circles; see below) and say, "what if we had a shape like this? How might we show equal size or 'fair shares' of each whole?" Provide students with multiple types of materials that they could use to show this, such as square tiles, scissors, pencils, etc. Be sure to allow students to select the materials they want to use to explore the task.

Allow time for pairs to explore. There are multiple ways that students may partition the shapes in half. Students may cover the left/right half of the shape if it is partitioned with a vertical line or top/bottom
half if the shape is partitioned with a horizontal line. Students may partition the shape in half with a diagonal line. Students also may fold the shape to make congruent halves or they may cut the shape in half. It may be that many students in the class use similar methods.

## O Look For

- Students who use an original method to partition the shape in half. This means that they use a method that few, if any, other students in the class are using. Ask the student to provide a justification for why this method helped them create a fair share. (Creative)
- Students who explore different materials for partitioning a shape in half. (Resourceful)


Students are engaged in the Encapsulation thinking skill during this discussion because they are starting to think about what it means to divide a shape in half.

## Explain

Come back together to share how the shapes were partitioned. Use a doc cam or "gallery walk" to have partners share how they partitioned the shape. Be sure to highlight the ideas such as the size of each piece has to be the same size. After each pair shares their shape, have students decide if the shapes were shared fairly or not. Have students tape their shared shapes on a chart labeled with two columns (Shared Fairly and Not Shared Fairly).

Ask students to look at the examples on the chart of the shapes that were shared fairly. Then, ask students what they noticed about the shapes when they were shared fairly. Have multiple students share ideas and be sure to follow up by asking students to explain why they think so (or what makes them think that). Ideas that students come up with might include that the pieces have to

- be the same size
- be the same shape (While a typical response, this is not necessarily true. Halves of the same whole do not have to be the same shape. The halves in this case $\square$ are the same as the halves in this case
- match (if they folded or cut the shape)
- have the same number of squares (if they covered it with square tiles)
Record students' ideas on a chart or the board.

During this discussion be sure at least one student mentions that when they created the two pieces of the shape to share many students made it so the pieces were the same size. Follow this discussion by asking students to think about what we call this, when we have two pieces of the same size from a whole. Students should be able to come up with the word, "half."

## Oook For

- Students who provide original or many ideas related to what they know about a half. (Creative)
- Students who recognize that there are multiple ways to partition a shape in half. (Perceptive)
- Students who begin asking questions about partitioning shapes into different numbers of pieces, 3 or 4 for example. (Curious)


## Elaborate/Extend

Target Task
Provide pairs with a different rectangle or circle (Shapes for the Elaborate/Extend section below) and ask students to partition the shape to be shared equally between four people.

Allow time for the pairs of students complete the task. As pairs are working ask if they notice anything about how they can partition the shapes into 4 equal shares. Have students document the ways that they can partition shapes in half and/or in fourths. This can be done with the shapes (cutting, covering, drawing lines on, etc).

Extend the Task
For students who were observed engaging in one of the highpotential behaviors or demonstrated advanced understanding of the concept in the Explore or Explain sections of the lesson, this target task can be extended in the following ways:

- Provide students the Shapes for Elaborate/Extend. Encourage students to look for patterns in the ways they can partition the shapes in fourths. For example, students might notice that they can make four equal shares by partitioning the halves in half. Ask students if this will work for all shapes (they can use
the Shapes for Tiered Option to investigate this). Have students explain their pattern to the whole class during the Evaluate discussion.
- Provide students with the Shapes for Tiered Option below and ask students to determine which can be partitioned in equal fourths. (It will not work for the half-oval shape.) Partitioning shapes such as these would be considered beyond what is expected for the grade level standard.


## Scaffolding and Support

For students who may need some support determining how to partition a shape in equal fourths, provide students with the square from the Engage section to work with at this time. Encourage students to use what they know about how to partition the square in half to partition it into equal fourths since they already have successfully partitioned this shape in half and discussed it with the class. Students could also be encouraged to fold the shapes to make sure the resulting parts match or are the same size.

## Look For

- Students who notice and share that they can make four equal shares by partitioning the halves in half. (Perceptive)
- Students who are using multiple or original methods to partition the shapes. (Creative)
- Students who partition shapes in fourths in unique ways. For

- Students who are able to clearly articulate what they did and/or how to partition the shapes into equal fourths. (Communicative)


## Evaluate

As pairs are wrapping up their investigation, bring the class back together. Ask each pair to explain how they investigated the problem and what they learned about representing one-fourth.

Some prompts that could be used during this part of the lesson are:

- What are some of the ways that you split the shape into fourths? (Have students show examples)
- Why is it that some of the ways we made fourths look different?
:
Engage students in Encapsulation during this discussion by having students explain the most important thing to remember about fairly sharing or partitioning a shape.

Wrap up the discussion by posing this question - What is the most important thing to remember about how to make fair shares for shapes? Have multiple students share their ideas during this discussion. Ultimately, you want students to develop the idea that when partitioning shapes into equal shares the resulting parts must be the same size. In first grade, students may still think that the parts must also be the same shape. However, as some students might discover in the Elaborate/Extend, the parts could be different shapes but still be the same size.

## Sandwich for Engage



## Shapes for the Explore Task

$\square$
$\square$

## Shapes for the Explore Task



Shapes for Elaborate/Extend


Shapes for Elaborate/Extend



Shapes for Tiered Option


Shapes for Tiered Option


Shapes for Tiered Option



## Lesson M2.1 School Shopping

In this lesson, students are provided with a multistep problem that involves determining a set of items to purchase given a set of constraints (spending limit). Students will need to explore a variety of item combinations to decide what they would purchase. This task also involves students in the Fluency, Originality, Flexibility, and Elaboration Thinking Strategy by having them think about multiple ways to solve the problem.

The activity section on page 67 of Math Matters describes some "Things to Think About" when teachers are observing students as they solve problems.

## CCSS.MATH.CONTENT.2.OA. 1

Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Standards for Mathematical Practice
MP1: Make sense of problems and persevere in solving them.
MP4: Model with mathematics.

Time Frame: ~ 2 math periods
To allow students to investigate the tasks and concepts in this lesson fully, it will likely take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered the day before during the Explain section of the lesson.

## Materials

School Supply Chart (electronic and hard copy)
Chart Paper and markers
Paper/pencils

## 7

When students are brainstorming here, they are engaging in the fluency part of this thinking strategy.

## Engage

Begin by telling students that the principal has contacted you and asked you to make a list of things you will need to buy for your classroom for next year. Ask them to help you brainstorm a list of the important school supplies we need for learning in the classroom.

Record students' ideas on the board or chart paper and encourage students to list as many ideas as possible and to build on other students' ideas. Brainstorm for about 3-5 minutes.

Tell students that this is a great list of materials and that the principal also shared a list of materials that the students could choose from. Share the school supplies chart with the students (projected on screen or hang chart on board). Ask students what they notice about the items on the list. For example, students might identify items that are similar or different from the list they created.

Explain to students that their job for the day will be to decide which school supplies they want to buy for the classroom. Ask students what are some important things to remember when shopping. One idea to encourage students to come up with is that they need to think about how much money they have to spend. Tell students that we call this amount of money that we have to spend a budget. Ask several students to share what they think budget means.

## Explore

Project/post and read the following problem to the class -
You and your partner have $\$ 50$ to buy school supplies for our class! Your job will be to decide which items you can purchase.

Ask students to talk with their partner about what they think the problem means. Have pairs share out with the class their ideas about what they are trying to solve with this problem.

Project/post and read the two additional pieces of information for the problem:

- You need to buy at least three different items. (You can buy more.)
- Try to spend as much of the $\$ 50$ as you can!

Tell students that they should work with their partner to figure out what to buy and how much it will cost. They should also be ready to explain their work to the class. Provide partner pairs with chart paper to record their work. Tell students that they need to include the following on their chart paper:

- items you are buying,
- cost of each item,
- a representation of your work *(see side note),
- and the total cost.
*Keep in mind: Students' work representations may look different for different groups. For example, students might use number lines, drawings of base ten blocks, equations, or nonstandard/standard algorithms. All of these are valid ways to show work for this problem, as well as ways for students to show flexibility in thinking.

As students are completing and recording their work, observe to see that students are meeting the criteria for the task (at least 3 items, spend as much of the $\$ 50$ budget as possible), and recording all necessary information.

## Olook For

- Students who are able to find multiple alternative solutions to the problem. (Strategic, Creative)
- Students who are organized in their mathematical work. (Strategic)
- Students who are thoughtful about prioritizing items to purchase. (Resourceful)


## Explain

Hang up each group's chart paper and have students do a gallery walk to look at the work of their classmates. Tell students they should look for things that are similar and things that are different about how other groups solved the problem.

Bring the students back together in a class meeting spot and ask them to share some of the similarities and differences that they noticed. Encourage them to use "adding on" language to show how they are connecting with what they hear from one another. Record student responses on white board or chart paper. Highlight that students used many different strategies and had different ways of choosing their items to fit the constraints of the problem.

## OClook For

- Students who express and expand their ideas in detailed or organized ways. (Communicative)
- Students who provide clear and exhaustive explanations of their reasoning. (Communicative)
- Students who demonstrate solving the problem in different ways than others. (Creative)


## Elaborate/Extend

In this part of the lesson, students will continue to work on the school supply problem; however, the constraints of the problem will change to increase the complexity of the task. Students will work with a partner during this part of the lesson. Pair students who demonstrated similar high potential behaviors during the Explore part of the lesson. Students who demonstrated these behaviors could work on a modified problem (described in the Extend the Task section below).

Target Task
Pose the adaptations to the problem by telling the students:
The principal contacted me last night, and said he wants to make sure that we have enough school supplies, so our budget has been increased to $\$ 100$.
Ask students how this changes the problem. Have a few students share ideas. Tell students that like yesterday/in the earlier part of the lesson, they will record work on chart paper and can use any strategy to solve the problem.

Provide partner pairs with chart paper to record their work and remind students that they need to include the following on their chart paper:

- items you are buying,
- cost of each item,
- a representation of your work*,
- and a total cost.

Extend the Task
For students who were observed engaging in one of the highpotential behaviors or who demonstrated advanced understanding of the concept in the Explore or Explain section of the lesson, this target task can be extended. Tell students that the principal also wants to make sure that there are enough materials for all of the students in the class. Explain that they should use the school supply list to select three items from the list that every student in the class would need. Have students determine how much it would cost to buy one for everyone in their class. Alternately, you could tell students they have to do this but keep the cost within $\$ 100$, so they have to identify
which items they could buy for everyone and still keep within that budget.

Scaffolding and Support
If pairs seem to need more support, consider making suggestions or asking scaffolding questions. For example:

- How can you use what you know from solving the problem with $\$ 50$ to solve the problem with $\$ 100$ ?
- Explain how you can keep track of the items on your chart. Be sure when scaffolding that students still try to figure out how to solve the problem without being told directly.


## OClook For

- Students who identify many ways to organize the items into sets that will total $\$ 100$. (Creative)
- Students who efficiently apply problem solving strategies. (Strategic)
- Students who notice the pattern that if they double all of the items bought in the Explore task, the cost of all the items in this task will double and still be within the $\$ 100$ limit.
(Perceptive)


## Evaluate

As pairs are wrapping up their work, bring the class back together and hang up the charts of their work. Have students take a gallery walk to look for similarities and differences in the items the students bought and the ways they solved the problem. The charts from the pairs that solved the Extend task can be included in this gallery walk.

Bring the students back together in class meeting spot and ask students to share some of the similarities and differences that they noticed. Record student responses on white board or chart paper. Highlight that students used many different strategies and had different ways of choosing their items to fit the constraints of the problem.

| School Supply List |  |  |
| :---: | :---: | :---: |
| Item |  | Cost |
|  | Pencil | \$1 |
|  | Box of markers | \$4 |
|  | Pack of Crayons | \$3 |
|  | Classroom Library Book | \$15 |
|  | Class ebook Reader | \$55 |


|  | Backpack | $\$ 42$ |  |
| :--- | :--- | :--- | :--- |
|  |  | Headphones |  |
|  |  |  | $\$ 14$ |



## Lesson M2.2 <br> Arrays Around Us

In this lesson, students will explore ways to use arrays to find patterns to support adding quickly and how to represent the total number of objects in an array using a repeated addition equation. Students are engaged in the Fluency, Originality, Flexibility, and Elaboration Thinking Strategy during this lesson when they are asked to develop multiple and/or unique (from other students in the class) equations that could represent the arrays provided. This introduction to arrays and repeated addition can serve as a foundation for the concept of multiplication.

For information about the interpretation of arrays as multiplication, see the Math Matters book (p. 76-80).

## CCSS.MATH.CONTENT.2.OA.C. 4

Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Standards for Mathematical Practice
MP7: Look for and make use of structure.
Time Frame: ~60 minutes
To allow students to investigate the tasks and concepts in this lesson fully, it will likely take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered the day before during the Explain section of the lesson.

## Materials

2 Twelve dot pictures (see below)
Picture of hearts in an array (see below)
Square sticky notes
Square inch tiles
Graph paper
Word wall cards (row, column, array)
Video (link included)
Copies of the Wall Tile Arrays
Copies of the Exit Ticket
Stickers (optional, for the exit ticket)

## Engage

The teacher will begin the lesson by showing the students two different pictures of 12 dots (see example) one at a time. Start with showing the picture of the 12 dots scattered and asking students, "How many dots do you see in the picture?" and "How do you know?" Have several students answer.


Then show students the picture of the 12 dots arranged in an array and asking, "How many dots do you see in the picture?" and "How do you know?" Have several students answer.


Facilitate a discussion about with which arrangement/picture it was easier to figure out the number of dots. Questions the teacher might ask include, Which of these pictures is easier to count? How are the pictures different? Why is it easier for you to find the total number of dots in one picture versus the other? etc. During the discussion teacher first may have students share with a partner and then share out with the whole class. Be sure to have multiple students share ideas throughout the discussion. Students should ultimately conclude that the dots in the array arrangement are easier to count and they may provide multiple reasons for why. Some students might mention the idea that they can add the number in the row/columns.

## 9

When students are thinking about multiple possible equations for representing the array, they are engaging in thinking fluently.

## Explore

Show the students a new set of images (see below) arranged in an array and ask students to think about how they might show the total number of hearts using an equation (or number sentence). Ask students if they think there might be more than one equation they could write to show the total number of hearts.

## 1

Questions such as these encourage students to engage in fluency, flexibility, and originality of thinking.


Provide each student with a picture of the heart array and have students work with a partner to develop as many equations that show the total number of hearts as they can. At this point students may develop number sentences that have equal addends (and that represent the row and columns, ex. $4+4+4+4+4=20$ ) or they may use unequal addends (may circle groups of hearts with different numbers and represent the equation to match, ex. $8+8+4$ ). Have students record their equations on sticky notes; they should only record one equation on each sticky note.

Throughout the student work time, encourage students to develop multiple equations that can be used to represent the total number of hearts in the array. The teacher might say, "How many different ways can you use addition to figure out how many hearts there are? Can you find a way that no one else will?"

## OOLook For

- Students who are using addition strategies. (Strategic)
- Students who are able to explain repeated patterns and connect them with arrays. (Communicative)


## Explain

In reference to the equations students developed to represent the array of 20 hearts, have pairs of students share the sticky notes with their equations. As each pair shares, have students sort or organize the sticky notes on a chart or white board by grouping equations that have equal addends together and equations that have unequal addends together. Although not expected at this time, if students used multiplication equations to represent the arrays they also can be grouped together.

Once all pairs have shared, call students' attention to the equations that have equal addends. Tell students that in mathematics we say that equations like this are showing repeated addition because the addends in a given equation are the same so the addends are "repeating" (ex. $4+4+4+4+4=20$ or $5+5+5+5=20$ ).

Consider creating word wall cards for the words row, column, and array. These can be introduced as they are discussed in this part of the lesson.

This may be a brief discussion at this point depending on students' prior knowledge/ experience with arrays. If they struggle to come up with some, show the video and then discuss arrays that they might see at school or at home.

## (1)

This part of the lesson may help students with flexibility in thinking as they consider that arrays can be many different things.

At this point in the lesson, introduce the word "array." The teacher can say, "An array is an arrangement of pictures, objects, or numbers in columns and rows." The word "array" and the definition can be added to a word wall that students use. Students could eventually add examples of arrays to the word wall. Continue the description of an array by describing the rows and columns. The teacher can say, "A row goes across (teacher directs students to move their arm horizontally in the air). A column goes up and down (teacher directs students to move their arm vertically in the air)."

Ask students to talk with a partner about where they might see arrays around school or home. Allow partners to discuss ideas and then have pairs share out ideas to the class. Be sure to solicit many ideas from different pairs in the class. Record students' ideas on a chart or whiteboard.

Follow this discussion by asking students how a repeated addition equation can be used to represent the total number of objects in an array. Encourage students to use to hearts array from earlier in the lesson to justify their ideas. Show students other examples from this video - http://viewpure.com/ks-q6gKoQKs . Pause after each array so students can see and discuss the repeated addition equations. The teacher can say, "Look at each row, the number for the items in each row is used in the number sentence." (Ex: "In the first picture we see a carton of eggs. The first row has 6 eggs and the second row has 6 eggs. If I add them together, $6+6=12 .{ }^{\prime \prime}$ )

Continue the discussion of the arrays and equations. When the picture of the soda cans appears ( 40 second mark), pause the video and have students discuss the possible equations for total number of cans in the array. The video contains many more examples of arrays with small and large numbers of objects.

## OClook For

- Students who recognize different ways to represent the arrays. (Perceptive)
- Students who connect background knowledge to everyday arrays. (Resourceful)


## Elaborate/Extend

## Target Task

In the previous part of the lesson, students may have said that some school walls or bathroom walls have arrays. Tell students that you have pictures of three tile walls (see below). Use one of the arrays to demonstrate how students might record what the array looks like on graph paper and to write the repeated addition sentence to represent it. Have partners select one of the wall tile arrays (1-3) to investigate other arrays that could be used to represent the same number of tiles. Students can use square inch tiles to represent the tiles in the array so they can manipulate the tiles to make arrays. For example, Wall Tile Array 2 represents 36 tiles as 4 rows of 9 tiles. Students would work to come up with other arrays that represent 36 , such as 6 rows of 6 tiles, 3 rows of 12 tiles, 12 rows of 3 tiles, etc. Students can record the arrays they make on graph paper and should include the repeated addition sentences that would represent the array they made. Note that when the students write the equations, the addends that are repeated can be either the number of tiles in the rows or the number of tiles in the columns.

## Extend the Task

For students who were observed engaging in one of the high-potential behaviors or demonstrated advanced understanding of the concept in the Explain section of the lesson, this target task can be extended by providing students with the Wall Tile Array for Extend the Task modification which represents 72 tiles. Have partners investigate other arrays that could be used to represent the same number of tiles. For the number 72 there are more arrays that students could create. Students can use square inch tiles to represent the tiles in the array so they can manipulate the tiles to make arrays. Students can record the arrays they make on graph paper and should include the repeated addition or multiplication (if they come up with them) equations that would represent the array they made.

Scaffolding and Support
If pairs seem to need more support, consider assigning an array that has fewer tiles than the others, like Wall Tile Array 1, which represents 30 tiles. Also, since each of these wall tile arrays is beyond what the CCSS standard specifies ( $5 \times 5$ arrays), the pictures can be cut to be $5 \times 5$ arrays or smaller if some students are still developing the meaning of the array. Students can record the arrays they make on graph paper and should include the repeated addition sentences that would represent the array they made.

## Ontook For

- Students who recognize different ways to represent the arrays. (Perceptive)
- Students who connect background knowledge to everyday arrays. (Resourceful)


## Evaluate

Using the fishbowl sharing out strategy, have all the pairs of students who used the same wall tile array sit on the inside of a circle. Have the rest of the students stand around the outside of the circle. The students in the center of the circle will share their repeated addition equations. Ask the students around the outside of the circle what they notice or what questions they want to ask. Conduct this fishbowl discussion with the students who investigated each wall tile picture having a turn in the center of the circle.

As an exit ticket, students will think of an array that they may see in school or at home and draw it. They should also then write at least one repeated addition equation that matches the array. If students struggle with keeping rows/columns evenly spaced by drawing the arrays, consider providing stickers for students to use instead of drawing the arrays.

Engage - Arrangement of 12 dots


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## Explore



Wall Tile Array 1 (30 tiles)


Wall Tile Array 2 (36 tiles)


## Wall Tile Array 3 (48 tiles)



Wall Tile Array for Extending the Task modification (72 tiles)


Name:

## Exit Ticket

Think about the video we watched. We noticed that arrays are everywhere in our world. Think of an array that you have seen and draw an example. Write at least one repeated addition equation that matches the array.


Equations that match the array:





## Lesson M2.3

## 3-Digit Place Value

In this lesson, students will explore ways to make three digit numbers given three digits. Students will use the point of view thinking strategy as they evaluate the digits they select and determine how they can create a larger number in comparison to another number.

For information about place value in our base-ten number system see the Math Matters book (p. 25-30).

## CCSS.MATH.CONTENT.2.NBT.A. 1

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.

## Standards for Mathematical Practice

MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.

Time Frame: ~ 60 minutes
To allow students to investigate the concepts in this lesson fully, it may take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered during the Explain section of the lesson before moving into the Elaborate/Extend section.

## Materials

0-9 Dice (3 per pair)
3-Digit Number Handout (Explore)
Make a Larger Number recording sheet (one per student)
Create the Target Number recording sheet (one per student)
$0-9$ Digit Cards (1 set per pair of students)

## Engage

Display the following image of " 57 " for all students to see. Ask students to think about the number being represented, and different ways they can represent it.


After students have time to think, instruct them to share their thinking with a partner.

Call on 2-4 students to share their thoughts with the whole group. Highlight the differences in thinking, explaining that there are many ways to represent a number.

## Explore

In this part of the lesson, students will work with a partner to compare threedigit numbers that they create by rolling three 0-9 digit dice. Provide each student with the 3-Digit Number handout and three 0-9 digit dice.

Explain the game to the students and then model one round of the game. This can be done as the teacher vs. the class, or one-half of the class vs. the other half of the class. Either way, be sure to model how to record the results on the 3-Digit Number handout. The directions for the game are as follows:

- Each student in the pair will roll the three dice, determine the largest three-digit number they can from the digits rolled, and write the digits on the three-digit number handout.
- Students then will compare their number with their partner. Have the students decide which number is bigger. Tell students they need to prove how they know which number is bigger. (They can draw or write an explanation.)
- Students will repeat this task four times and mark each turn on the record sheet.

Provide time for the students to play the game with a partner.

## OClook For

- Students who are able to provide more than one explanation (creative)
- Students who articulate clear explanations (communicative)


## Explain

Bring students together and have pairs share their numbers and explanations. You may have students show their ideas using a document camera so all students can see them. Draw students' attention to different explanations or representations.

Some questions that could be asked to facilitate this discussion are as follows:

- How did you decide which partner had the larger number?
- If our task was to find the smaller number instead of larger, how would this change our point of view on what strategy to use?

Highlight ideas that provide different representations or how students represented their numbers to show how they proved that the number was either larger or smaller than their partner's number.

## OuTlook For

- Students who are able to provide more than one explanation (creative)
- Students who articulate clear explanations (communicative)


## Elaborate/Extend

Tell students that they are going to continue exploring 3-digit numbers in the next task.

Target Task
Pass out three 0-9 dice and one set of 0-9 digit cards to each pair of students. Also provide one Make a Larger Number recording sheet to each student.

Explain the game to the students and then model one round of the game. This can be done as the teacher vs. the class, or one-half of the class vs. the other half of the class. Either way, be sure to model how to record the results on the Make a Larger Number recording sheet. The directions for the game are as follows:

- The pair will roll the 3 dice once and determine the smallest threedigit number that can be made from the digits rolled. Each student should record this number on the Make a Larger Number Recording sheet. This will be the Starting Number.
- Students next shuffle the pile of 0-9 digit cards and take turns selecting one card at a time until each student has 3 digit cards.
- Students take turns making numbers from the three digits they selected that are larger than the Starting Number. Students should record each number they create that is larger than the Starting Number. The student who can make more numbers that are larger
than the Starting Number wins that round.
- Example round: Students roll the dice and get the digits 6, 5, and 2. They write the number 256 on the recording sheet. Students take turns selecting cards. Student A gets the cards 4, 9, and 1, and Student B gets the cards 7, 3, and 5.
- Student A writes 491
- Student B writes 735
- Student A writes 941
- Student B writes 357
- Student A writes 419
- Student B writes 537
- Student A writes 914
- Student B writes 573
- Student A passes (no more larger numbers)
- Student B writes 753 and wins the round.
- Students can repeat the game for several rounds.

Extend the Task
For students who showed high-potential behaviors or who demonstrated advanced understanding of the concept in the Explore/Explain sections of the lesson, this target task can be extended in this way:

- Have the students roll the three 0-9 dice and determine the largest three-digit number that can be made from the digits rolled.
- From the shuffled pile of digit cards, students will select 5 cards each. (There will be no digit cards left in the pile.) Alternatively, if students are working in groups of 3 , provide each student with a set of 0-9 digit cards, and students will shuffle the cards and select the top five digit cards to use for that round.
- Students will use their digit cards to create an addition equation that will result in a sum that is as close to the three-digit number they rolled as possible. The student in the pair with the closest sum to the target three-digit number wins that round. Note that the students could create addition equations that could have two or more addends, use at least 4 (but not all 5) of their digit cards, or have addends with different numbers of digits (ex. A 3-digit number + a 1digit number + a 1-digit number).
- Example game round: Students roll the dice and get the digits 4, 6, and 3. They write the number 346 on the Create the Target Number sheet. Students select their cards; Student A has 2, 3, 5, 7, and 8, and Student B has 1, 4, 6, 9, and 0.
- Student $A$ arranges cards to create this equation: $325+7+8=$ 340
- Student B arranges cards to create this equation: 196+40 = 236
- Student A wins the round.

For a further extension of this, you could allow students to use addition and subtraction operations in their equations.

## Scaffolding and Support

Consider offering manipulatives to support struggling students Use any classroom support anchor charts/posters about place value to support students' thinking.

If students seem to need more support with coming up with a way to represent their number, consider asking the following questions to prompt student thinking:

- How can you use the manipulatives to help you show your number?
- What picture can you draw that might help you?
- What do you know about this number?


## O-OLook For

- Students who apply strategies from previous lessons (strategic)
- Students who are able to come up with several different numbers (creative and resourceful)
- Students who continue to persevere through the task (resilient)
- Students who are able to come up with multiple numbers with their digits (Strategic)
- Students who articulate clear explanations (communicative)


## Evaluate

Bring the students to the discussion area and complete one round of the Target Task as a class. (Roll the 3 dice to create the Starting Number, have each student select 3 digit cards to work with, and have them each try to create the largest number they can that is larger than the Starting Number).

Have students share out their largest numbers and determine who was able to create the largest number. Facilitate a discussion about how the students' point of view (or different digits they selected) influenced the outcome, or the largest number they could make. Encourage students to talk about ways to compare their numbers. Record these ideas on chart paper.

Or, you could do one round of the Extend the Task with the whole class and then facilitate a discussion about how the students' point of view influenced the ways they arranged the digits in their addition equations.

Name: $\qquad$ Date: $\qquad$
Roll the 3 dice and create the largest number you can. Compare your number with your partner.
Discuss who has the larger number.

|  | Hundreds | Tens | Ones | I had the larger <br> number <br> (Yes/No) |
| :--- | :--- | :--- | :--- | :---: |
| Turn 1 |  |  | $\mathrm{Y} / \mathrm{N}$ |  |$|$| Explanation or Representation |
| :--- |
| Turn 2 |


| Turn 3 |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Explanation or Representation |  |  |  |
| Turn 4 |  |  | $\mathrm{Y} / \mathrm{N}$ |

## Explanation or Representation

Name: $\qquad$ Date: $\qquad$

Record the Starting Number and all the larger numbers you can create with your digit cards.

| Starting Number |  |
| :--- | :--- |
|  |  |
|  |  |
|  |  |

Name: $\qquad$ Date: $\qquad$

Record the Target Number and the addition equation you created with the closest sum.

| Target Number | Addition Equation with the closest sum to the Target Number |
| :--- | :--- |
|  |  |
|  |  |




Grade 2 / Mathematics / M2.3 Lego Picture



## Lesson M2.4 <br> Picking Apples

In this lesson students will be provided with several word problems that have multiple solutions. The goal is to have students recognize that some problems can have multiple solutions and that problems can be solved using multiple strategies. Students will demonstrate and explain how they solve problems using a variety of strategies. Students will engage in flexibility and fluency to experiment with multiple addends or subtrahends (depending on how they solve the problems) that could make a problem true.

Please see pages 60-67 and 74 of the Math Matters book for more information related to solution strategies for solving addition and subtraction problems and observing how students solve problems.

> CCSS.MATH.CONTENT.2.NBT.B. 7
> Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
> Standards for Mathematical Practice
> MP4: Model with Mathematics

Time Frame: ~ 60 minutes
To allow students to investigate the tasks and concepts in this lesson fully, it may take longer than 60 minutes. If the lesson is to extend over two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to review briefly the conclusions students came to in the Explain section before they begin the investigation in the Elaborate/ Extend section.

## Materials

Chart paper/markers
Personal white boards and markers/clear sleeves
Record Sheets (regular and extend versions)
Manipulatives (as needed)
Digital/displayable versions of the four word problems throughout the lesson


Students demonstrate fluency when they can find more than one answer. Originality may appear when students find unique ways to solve the problem.

## Engage

Project or write the following problem with no numbers on the Interactive White Board (IWB) for all students to see:

There are lots of apples in the orchard. Your family comes to pick some apples. How many red and green apples did they pick?

Ask students, What do you notice about this problem? Give students time to think and then have them turn and talk with a partner about what they noticed. Ask several students to share out their ideas about what they noticed about the problem. As students share ideas, jot down their ideas on a chart or the board.

Additional teacher prompts that can be asked during this time to help facilitate the discussion are as follows (use as needed): What do we know? What else do we need to know? What information is missing? Was there any information that you did not need? What questions do you have?

During this discussion look for students to recognize the following ideas:

- The context of the problem is that the family is picking apples..
- The problem is asking about how many apples there are and how many get picked.
- There is not enough information presented to actually solve the problem.

Explain to students that it is important to make sense of a problem before trying to solve it and that good problem solvers will ask themselves questions such as those that were discussed before beginning to solve the problem or creating number sentences.

## Explore

Project and discuss the following word problem with the students: Your family goes to the apple orchard to pick red and green apples. Farmer Brown, who owns the orchard, tells you to pick 20 apples. How many red and how many green apples could you possibly put in your basket?

Encourage students to consider the following questions during the discussion:

- What do you notice about this problem?
- Could there be more than one answer? How do you know?
- What strategies might we use to solve the problem?

Ask the students to work on and solve the problem on their whiteboards in a way that demonstrates their strategy. Circulate the classroom and


Students are using flexibility as they engage in understanding multiple solutions and/or strategies.
observe the strategies that students are using to solve it.

## Olook For

- Students who recognize that there is more than one answer to a problem. (Strategic)
- Students who recognize that there is more than one strategy to solve the problem. (Strategic, Creative)
- Students who recognize patterns such as if I increase by one for one color, I can decrease by one for the other color or grouping by 5, 10, 2. (Perceptive)


## Explain

Have students choose a partner. Ask groups/partners to compare the number of each color of apples they used to make their total and to compare the strategies they used to solve the problem. Questions might include: Did you all have the same number of red and green apples? Explain why you may or may not have different numbers of red and green apples. How were your problem-solving strategies the same? How were they different?

You are looking for students to recognize that there can be more than one solution to the problem and multiple strategies can be employed to get there. You want students to recognize these strategies for possible use in the next problems.

Facilitate a discussion to have several groups share out their strategies to show that there are a variety of ways to solve the same problem. Questions that could be asked (as needed) are:

- What did you notice about how you solved the problem?
- What did you notice about your answers to the problem?
- What was the same?
- What was different?


## O- Look For

- Students who recognize that there is more than one answer to a problem. (Strategic)
- Students who recognize that the problem can be solved with multiple strategies (e.g., picture, tens and ones, words, number line, tape diagram, expanded form /decomposing) (Perceptive)
- Students who communicate their thinking clearly. (Communicative)
- Students who recognize patterns such as if I increase by one for one color, I can decrease by one for the other color or grouping


Students demonstrate fluency when they can find more than one answer. Originality may appear when students find unique ways to solve the problem.

```
by 5, 10, 2. (Perceptive)
```


## Elaborate/Extend

Target Task
Students will work on solving a problem that builds on the Explore task as they now work with three colors of apples. Project and have students discuss the following problem:

There are 25 apples in a basket. 15 are red, some are yellow, and some are green. How many apples of each color can be in the basket? How many solutions can you find? Explain your thinking.

Students can be encouraged to use the questions generated earlier in the lesson to discuss this problem.

Provide students with time to work on the problem with a partner or independently (depending on the class/student preference) and record their answers/strategies using the record sheet. Observe for and note the different strategies students use to solve the problem.

Extend the Task
For students who were observed engaging in one of the high-potential behaviors or demonstrated advanced understanding of the concept in the Explore/Explain section of the lesson, this target task can be extended by having the students solve a more complex version of the Target Task.

There are 36 apples in the basket. Some are red, some are yellow, and some are green. There are more red apples than green apples, and more green apples than yellow apples. How many of each color can be in the basket? Show your thinking.

Note that this is not to be done in addition to the Target Task, but rather in place of it. Provide students with time to work on the problem with a partner or independently (depending on the class/student preference) and record their answers/strategies using the record sheet.

Scaffolding and Support
If students seem to need more support with solving these types of problems consider asking the following questions to prompt student thinking:

- What information do you know?
- What do you still need to know?
- What strategies have we learned today that might help us?
- What has to be true about these numbers?


Students are able to elaborate on different strategies used to find more than one solution to the same problem.

- How might manipulatives or pictures help you to solve?


## OOLook For

- Students are able to find multiple solutions using patterning. (Perceptive/ strategic)
- Students are able to come up with multiple solutions. (Strategic)
- Students use sample problems or problems from the lesson to complete their problem. (Resourceful)
- Students are able to explain their thinking. (Communicative)


## Evaluate

Use a gallery walk to share out strategies. Bring students back to the carpet to share their observations during their gallery walk. Note that even students working on the Extend the Task problem can join in this discussion because it is about the strategies they used, not just the solutions. Some questions may include: What strategies did you notice? Did anyone come up with more than one strategy? How did you come up with multiple solutions/ strategies?

Again, you are looking for students to recognize that there can be more than one solution to the problem and multiple strategies can be employed to get there. Not all math problems have only one solution or one way of getting to the solution.

## Picking Apples - Target Task Record Sheet

Name: $\qquad$

Solve. Be sure to show your thinking and explain.

There are 25 apples in the basket.
15 are red, some are yellow, and some are green.
How many apples of each color can be in the basket?
How many solutions can you find? Explain your thinking.

Name: $\qquad$

Solve. Be sure to show your thinking and explain.

There are 36 apples in the basket.
Some are red, some are yellow, and some are green.
There are more red apples than green apples, and more green apples than yellow apples.
How many of each color can be in the basket?
Show your thinking.


## Lesson M2.5

## A Trip to the Movie Theater

In this lesson, students will explore the idea that math is used in many situations outside of school and that there are questions we can ask ourselves when solving problems. Students will engage in the questioning thinking strategy as they generate questions to progress through the problem solving process.

Please see pages 60-67 and 74 of the Math Matters book for more information related to solution strategies for solving addition and subtraction problems and observing how students solve problems.

## CCSS.MATH.CONTENT.2.NBT.B. 7

Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Standards for Mathematical Practice
MP1: Make sense of problems and persevere in solving them.
Time Frame: ~60 minutes
To allow students to investigate the tasks and concepts in this lesson fully, it may take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered the day before during the Explain section of the lesson.

## Materials

Chart Paper/Markers
Movie Theater Problem Slides (in Google folder)
Movie Theater Explore Problem hand out
Movie Theater Elaborate/Extend Problem handout
Movie Theater Extend the Task Problem handout

## Engage

Begin by gathering students in a class discussion space. Display a blank
tally chart with the title, Do you use math outside of school? and two columns Yes and No. Ask students to think about this question and then have each student record a tally under the column for his/her response (Yes/No).

Once all students record their responses, ask them to think about what they notice about the responses on the chart. Have several students share what they noticed about the data on the tally chart.

After students share, ask the students who answered yes, "When is a time that you used math outside of school?" Alternatively, this could be done by having students jot their ideas down on sticky notes to post and share. After providing students with time to think (jot down) their ideas, have students share their ideas.

## Olook for

- Students who suggest "unconventional" uses of math outside of school (i.e. using measurement when cooking, sharing food). (Creative)
- Students who come up with many and varied uses of math outside of school. (Creative)
- Students who make connections between other students' uses of math outside of school. (Perceptive)

Students engage in questioning here by generating questions about the situation.

## Explore

Tell students that sometimes we use math outside of school when we go to different places, like to the movie theater. Project the Movie Theater Problem slides. Show students the picture of the movie theater seats on SLIDE 1. Ask students, "What do you notice? What do you wonder?" Give students time to think and then ask them to share out. Record student "noticings" on chart paper as their ideas are shared.

Display the picture of the movie theater with the situation on SLIDE 2: Some 2nd grade classes are going to the movies to see _(current movie). Display SLIDE 3 and ask students: "What math questions could we ask about this situation?" Ask students to share their questions and record questions for all students to see (on chart paper/board).

As a class select a target question. Lead students (but do not tell them directly) to the question - Are there enough seats for all the 2nd graders to sit in the theater? Ask students to go ahead and solve the problem (knowing that they do not have enough information to solve it).

Students engage in questioning here by generating questions about the information they need to solve the problem.

Once students realize and express that they do not have enough information to solve the problem, display SLIDE 4 and ask What questions do you have about the information we need?" Give students time to think and then pair-share their ideas.

Ask volunteers to share their ideas of what information is needed to solve the problem. Show SLIDE 5 to reveal the missing information from the problem (number of classes (3), number of students in each class $(\mathbf{2 1}, \mathbf{2 0}, \mathbf{2 1})$ number of seats in the theater (100)).

Ask students if they now have enough information to solve the problem. Once they express that they have enough information, pass out Movie Theater Explore Problem hand out. Allow time for students to solve the problem independently or with a partner.

## OOLook for

- Students who come up with multiple questions related to the problem. (Curious)
- Students who can analyze the problem to identify one or more pieces of missing information. (Strategic)
- Students who can communicate what is missing with their partner, as well as why it is important. (Communicative)
- Students who are solving the problem using a specific strategy (e.g., drawing a picture, finding tens). (Strategic)
- Students who use more than 1 strategy to solve the problem. (Perceptive)
- Students who use unique strategies to solve the problem (math strategies that have not been covered yet). (Creative).


## Explain

When students have completed the problem, have each group/student share out if there are enough seats for the three classes to fit in the theater. This can be recorded using a new tally chart or just by recording the number of yes and no answers. Have several students share the strategy they used to solve the problem. Ask students to evaluate if that is a strategy that would work for this problem. Note that there are many different strategies students could use to solve this problem. Be sure to have multiple students/partner pairs share their ideas so that multiple strategies are discussed.

After sharing strategies, have students come to a conclusion about if there were enough seats in the theater for the three classes. Also,

Students engage in questioning here by generating questions about the information they need to solve the problem.
comment on the real-world application of multiple math strategies in this problem.

## Or Look for

- Students who can communicate the strategy they used and explain why it works. (Communicative)
- Students who can identify the additional information of how many seats are left in the theater. (Curious)


## Elaborate/Extend

Target Task
Students will work on solving a problem that builds on the Explore task as they are provided with additional information. Display SLIDE 6 for students: Also, two third grade classes will go to the movie. Will they all fit in the theater now?

Ask students to go ahead and solve the problem (knowing that they do not have enough information to solve it). Once students realize and express that they do not have enough information to solve the problem, ask them to generate questions about what information is needed. Once they identify that they need to know the number of students in the third grade classes, display SLIDE 7, which identifies that each third grade class has 23 students.

Ask students if they now have enough information to solve the problem. Once they express that they have enough information, pass out Movie Theater Elaborate/Extend Problem hand out. Allow time for students to solve the problem independently or with a partner.

Extend the Task
For students who were observed engaging in one of the high-potential behaviors or demonstrated advanced understanding of the concept in the Explore/Explain section of the lesson, this target task can be extended by having the students solve a more complex version of the Target Task. Explain to these students that they moved the classes to a theater with 150 seats and that now they have to figure out how many seats will be empty. Provide these students with the Movie Theater Extend the Task Problem handout. ${ }^{* *}$ Note that this problem should replace the Target Task, not be done in addition to it. Provide students with the time to work on the problem with a partner or independently (depending on the class/student preference).

Students engage in questioning here by generating questions they could ask throughout the problem solving process.

Scaffolding and Support
For students who seem to need support in solving the problem prompt them with scaffolding questions:

- What strategy that we already know (from the first problem) could we use to solve this problem? (i.e. drawing pictures, baseten blocks, decomposing)
- What is the question asking us to figure out?
- Ask students which manipulatives might be helpful.


## O Look for

- Students who are solving the problem using a specific strategy (i.e. drawing a picture, finding tens). (Strategic)
- Students who use more than 1 strategy to solve the problem. (Perceptive)
- Students who use unique strategies to solve the problem (math strategies that have not been covered yet). (Creative)


## Evaluate

When students have completed solving the problem, have them bring their work to a class meeting spot. Have each group/student share out if there are enough seats for the three classes to fit in the theater. This can be recorded using a new tally chart or just by recording the number of yes and no answers. Have several students share the strategy they used to solve the problem. Ask students to evaluate if that is a strategy that would work for this problem.

Tell students that since all the classes cannot fit in the theater they moved the classes to a bigger theater with 150 seats and that some students had to figure out how many seats were empty in the new theater. Ask students who worked on the Extend the Task problem to share their solutions and strategies. Students could further extend the task by estimating how many other classes could be invited on the trip to the theatre.

To wrap up the lesson, revisit the idea that math is used outside of school in many ways and that when we have problems to solve there are lots of questions that we have to ask ourselves before we solve them. Ask students to think about the questions they had to ask throughout this lesson and give them time to share ideas with a partner. Have several students share out ideas about questions to ask when problem solving and record students' ideas on a chart/board. Students may come up with ideas such as what the problem is, what information they need, does the strategy/solution make sense, etc.

# Movie Theater - Explore Problem 

NAME $\qquad$

## A TRIP TO THE MOVIE THEATER!

Three second grade classes are going to the theater to see a movie.
Ms. Hale's class has 21 students.
Ms. Grande's class has 20 students.
Mr. Ash's class has 21 students.
There are 100 seats in the movie theater.
Are there enough seats to fit all of the students? Show how you know.

NAME $\qquad$

## A TRIP TO THE MOVIE THEATER!

Three second grade classes are going to the theater to see a movie.
Ms. Hale's class has 21 students.
Ms. Grande's class has 20 students.
Mr. Ash's class has 21 students.
Two third grade classes with 23 students each are going to the movie.
There are 100 seats in the movie theater.
Will they all fit in the theater now? Show how you know.

NAME $\qquad$

## A TRIP TO THE MOVIE THEATER!

Three second grade classes are going to the theater to see a movie.
Ms. Hale's class has 21 students.
Ms. Grande's class has 20 students.
Mr. Ash's class has 21 students.
Two third grade classes with 23 students each are going to the movie.
We moved to a theater with 150 seats to have enough seats.
How many seats will be empty? Show how you know.


## Lesson M2.6 Fun Friday

In this lesson, students will use the decisions and outcomes thinking strategy to make decisions about how to organize and represent data. The class will first generate data by taking a vote on potential Fun Friday activities. Later, the data will be combined with data from "other classes" to allow students to make decisions about how to organize and represent a larger set of data.

For information about representing data graphically, see the Math Matters book (p. 304-308).

## CCSS.MATH.CONTENT.2.MD.D. 10

Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Standards for Mathematical Practice
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.

Time Frame: ~ 60 minutes
To allow students to investigate the concepts in this lesson fully, it may take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered during the Explain section of the lesson before moving into the Elaborate/Extend section.

## Materials

$1 / 2$ inch graph paper (2/student)
Chart paper
Markers
Visual Fun Friday Charts (available here -
https://docs.google.com/presentation/d/1384wUveiGNbsqrrC8RCvXynyKMA R4G2TctMij5exYLU/edit?usp=sharing)
Copies of the Fun Friday Activity Votes (1 per pair)
Copies of the Fun Friday Activity More Votes (for students doing Extend the Task)

* A bigger version of this chart is attached, printed with materials, and available with the link in the materials section.


## Engage

Gather students in a space in the classroom that is conducive to discussions and pose this situation to the class:

The principal gave us some information about an upcoming Fun Friday. They had some other classes vote on different activities. But the principal is having trouble making a decision about the activities and would like us to help. Here is the information the principal shared with us.

Project this information for all students to see*:

| Fun Friday Choices |  |
| :--- | :--- |
| Outside games | 4 votes |
| Inside games | 3 votes |
| Both | 6 votes |

Provide students think time and allow them to talk with a partner about how we might help the principal make the decision and if there would be something we could create that would show the information in a different way. Give students think time and ask them to share with a partner. Then solicit ideas from multiple students and record these in a place that all students can see (chart paper, white board, etc.).

## Explore

Have students work with a partner to create a representation of the Fun Friday data that the principal shared. Students should have access to manipulatives (such as linking cubes), sticky notes, paper, markers, etc. The goal of this task is not that all students create the same representation, it's to allow students to explore different representations and/or representations that make sense to them.

## OClook For

- Students who develop multiple or unique ways to represent the data. (creative)
- Students who ask probing questions about ways to display the information. (curious)
- Students who make inferences about the data. (perceptive)


## Explain

Have each pair of students share their representation with the class. This can
be done through a gallery walk, by displaying students' representations through a document camera, etc. Ask students what they notice about the various representations.

Tell students that when mathematicians display data, they make decisions about the best type of representation to use that will make sense to others. One of the ways we can display data in a neat and organized way is by creating a graph.

Work together with the students to create a class bar graph on the white board or chart paper for all students to see. Be sure to guide students through this task by asking questions, instead of simply modeling how to do it. After creating the graph, facilitate a discussion about the data represented on the graph, such as:

- What can we learn from this graph?
- How many kids want to participate in each activity?
- How many more votes for one activity than another?
- What are some things you notice about the data?
- What are some things you noticed about the way the graph was designed?

Facilitate a discussion about the strategy of decisions and outcomes as it relates to representing data graphically. Some questions you could ask include these:

- How did we decide what labels to use on our graph?
- How did we decide what numbers to use on the $y$-(or $x$-) axis?
- What if we made different decisions (use an example from what the class did)? How might that change the way the graph looks?


## Olook For

- Students who think flexibly and see options for how the graph can be designed. (creative)
- Students who categorize in unique/multiple ways. (creative)
- Students who ask questions to help interpret their data. (curious)


## Elaborate/Extend

Display the Fun Friday Activity Votes* chart for all the students to see and tell students that the principal had classes vote again on which specific activities they would want to do for Fun Friday. Ask students what they notice about the chart and have students share their ideas.

| Fun Friday Activity Votes |  |  |  |
| :--- | :--- | :--- | :--- |
| Soccer | 4 votes | Basketball | 3 votes |
| Swings | 2 votes | Tag | 6 votes |
| Play uno | 3 votes | Draw | 3 votes |
| Computer time | 6 votes | Blocks | 4 votes |
| Board games | 2 votes | Dance | 5 votes |
| Playground | 7 votes | Chalk drawing | 3 votes |
| Jump rope | 2 votes | Play Go Fish | 3 votes |
| Read | 4 votes | Listen to music | 4 votes |
| Legos | 3 votes | Watch a movie | 3 votes |

Tell students that sometimes when making a graph we need to make decisions about how to organize the data before we make the graph. Ask students to think about and talk with a partner about how they could organize/sort the activities into categories. Have multiple students share ideas, and record the ideas on a chart or white board for students to reference as they are working. Some ways students might suggest that the data could be sorted are as follows:

- Indoor/outdoor activities
- Activities we do individually, with a partner, with a team
- Activities we need materials/equipment for, and those we do not
- Activities with a winner/loser, activities that do not have a winner/loser


## Target Task:

Provide each pair of students with a copy of the Fun Friday Activity Votes (see below). Tell students that they should cut out each of the activities and then sort them into categories. They need to decide within their pairs how they want to sort the activities. Have them work with their partner to decide on the categories that they will use.

Once students have sorted the activities into categories, they should create a graph to represent the data. Have each pair explain how they sorted their data to you and provide the graph paper to each pair as they are ready. Also be sure the sample, class-created graph is displayed for students to reference. Students will work with their partner to create their graph.

Extend the Task:
For students who were observed engaging in one of the high-potential behaviors or who demonstrated advanced understanding of the concept of decisions and outcomes in earlier sections of the lesson, this target task can be extended by asking students to consider a larger set of the data. This would result in larger numbers for students to consider how to represent graphically.

Provide students with the Friday Activity More Votes list. Tell students that they should cut out each of the activities and then sort them into categories. They need to decide how they want to sort the activities.

Provide students with graph paper and discuss with them how they might use a different scale on the Y -axis to represent the data on their graph. They should use the chart of questions from earlier in the lesson to make decisions about how to design their graph.

## Scaffolding and Support:

Provide additional support as students categorize the activities. As needed, provide scaffolding to students who need support with creating the graph. Graph paper with the $x$ - and $y$-axes already drawn on it may also provide additional support.

## Olook For

- Students who choose alternative ways to create their graph. (creative)
- Students who evaluate multiple ways to categorize the data or create the graph. (strategic)
- Students who discuss how a different decision would change the format of their graph. (perceptive)
- Students who clearly communicate their thinking about representing data or the decisions need to represent the data. (communicative)


## Evaluate

Have each pair display their graph (ex. on their desk, hanging them up around the room, etc.) Ask students what they notice about the graphs created by their classmates. They should notice that many of the graphs look different because the pairs categorized the data differently. Some questions you could use to guide this discussion include the following:

- What did you notice about the way the data is represented/graphed in all of the graphs?
- Did all our graphs look the same? Why or why not?
- How would your graph look different differ if you decided to sort the activities in a different way?

Wrap up the lesson by helping students understand that when creating graphs to show data, people have to make many decisions and that the decisions that are made affect the way the data are represented.

## Fun Friday Choices <br> Outside games 4 votes <br> Inside games 3 votes <br> Both <br> 6 votes

| Soccer 4 votes | Basketball 3 votes |
| :--- | :--- |
| Swings 2 votes | Tag 6 votes |
| Play Uno 3 votes | Draw 3 votes |

Computer time 6 votes Blocks 4 votes
Board games 2 votes Dance 5 votes
Playground 7 votes Chalk drawing 3 votes
Jump rope 2 votes Play Go Fish 3 votes

Read 4 votes
Listen to music 4 votes

Legos 3 votes
Watch a movie 3 votes

Friday Activity More Votes (Extend the Task)

| Soccer 9 votes | Basketball 7 votes |
| :--- | :--- |
| Swings 4 votes | Tag 5 votes |
| Play Uno 8 votes | Draw 9 votes |

Computer time 6 votes Blocks 8 votes
Board games 8 votes Dance 7 votes
Playground 6 votes Chalk drawing 7 votes

Jump rope 6 votes Play Go Fish 6 votes

Read 7 votes

Legos 8 votes
Listen to music 9 votes

Watch a movie 7 votes


Engage Section

| Fun Friday Activity Votes |  |  |  |
| :--- | :--- | :--- | :--- |
| soccer | 4 votes | Basketball | 3 votes |
| swings | 2 votes | Tag | 6 votes |
| Play uno | 3 votes | Draw | 3 votes |
| Computer time | 6 votes | Blocks | 4 votes |
| Board games | 2 votes | Dance | 5 votes |
| Playground | 7 votes | Chalk drawing | 3 votes |
| Jump rope | 2 votes | Play Go Fish | 3 votes |
| Read | 4 votes | Listen to music 4 votes |  |
| Legos | 3 votes | Watch a movie | 3 votes |



## Lesson M2.7 Is it half?

In this lesson, students will explore the idea that one-half of two identical wholes have to be the same size, but not necessarily the same shape. The Encapsulation Thinking Skill is exemplified in the discussion parts of the lesson, especially during the Explain and Evaluate sections of the lesson where students are discussing their ideas. Students should precisely explain their reasoning related to the idea that halves of wholes of the same size do not have to be the same shape.

For more information related to fractions as part of a whole, see the Math Matters book (p. 99-105 \& 114-119).

## CCSS.MATH.CONTENT.2.G.A. 3

Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

## Standards for Mathematical Practice

MP3. Construct viable arguments and critique the reasoning of others.

MP5. Use appropriate tools strategically.

Time Frame: ~ 60 minutes
To allow students to investigate the tasks and concepts in this lesson fully, it will likely take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered the day before during the Explain section of the lesson.

## Materials

Square sticky notes (3 inch squares)
Shapes to investigate (attached)
Square inch tiles
Chart paper/markers
Tape

## Engage

Pose the question, "What does one-half mean?" Allow students think time, and then ask them to share with a partner sitting next to them. Listen in to ideas that the students are sharing with one another and note ideas that should be shared with the whole class.

Upon bringing the class back together, have groups share out ideas about what one-half means. Allow for multiple interpretations and ask for examples. Record students' responses on chart paper or white board. Some ideas students might share are:

- Half means two equal pieces of a whole
- Half is breaking a shape in two pieces
- The two pieces have to be the same size

If students do not bring up the idea that the two pieces of a whole need to be the same size for one of them to represent one-half, be sure to ask questions to lead students to this idea before moving on with the lesson.

## Explore

Provide each student with one square sticky note (3 in. square). Ask students how they could show one-half of the sticky note. Allow students time to do so. Students might fold or cut the sticky note in half vertically, horizontally, or diagonally. They also might color in half of the sticky note. If a student only draws a vertical, horizontal, or diagonal line, be sure to ask them how they could prove that each piece is the same size. As students complete this task for one sticky note, provide another sticky note and ask if the student can determine another way to show one-half.

## O Look For

- Students who are precise in making the two pieces equal in size and shape. (Strategic)
- Students who are able to model one-half in more than one way. (Creative)


## Explain

Bring the class back together and have students share how they showed one-half with their sticky notes. As students share, have them tape their own examples to a chart. They should add their examples to other student examples that are similar (ex. those that folded or cut diagonally in the same group).

## $F$

Students are engaged in the Encapsulation thinking strategy during this discussion because they are starting to think about what it means to divide a shape in half.

Some questions that could be asked during this part of the lesson are:

- How does this model show one-half?
- What do we notice about all of the examples in this group? (point to the examples in one of the groupings) Emphasize that students should be specific in supporting their ideas.

The discussion should conclude with students' understanding the following points. Encourage several students to articulate and repeat these points.

- One-half of the square sticky notes can be shown in multiple ways,
- Each of the rectangles that result from folding/cutting the sticky note in half vertically or horizontally represents onehalf, as does each of the triangles that result when folding/cutting it in half diagonally, and
- The two rectangles are the same size and the two triangles are the same size.


## O Look For

- Students who are able to make connections between different examples. (Perceptive)
- Students who support their ideas by referring back to the examples. (Communicative)
- Students who articulate clear explanations. (Communicative)
- Students who begin to ask additional questions related to making halves and/or other fractional parts of a whole (Curious)


## Elaborate/Extend

Tell students that when they were explaining their ideas about showing one-half of the square you noticed that sometimes one-half was shown with a rectangle shape and sometimes with a triangle shape.

## Target Task

Show students the two squares attached below and ask them how it could be that both of them are cut in half when the shapes look different. Ask students to share ideas about how they could investigate and prove that the line in each shape divides it in half. Also, have students brainstorm materials they could use to investigate the problem (ex. students could use square inch tiles, sticky notes, cut the shapes, etc.).

Allow student pairs time to investigate the problem. As pairs are investigating, observe students' ideas and work. Make note of the ways students are investigating and look for evidence of understanding and high-potential behaviors.

Extend the Task
For students who were observed engaging in one of the highpotential behaviors or who demonstrated advanced understanding of the concept in the Explain section of the lesson, this target task can be extended by asking students to investigate how different shapes can all represent one-fourth of the shape instead of one-half (see the Shapes for Extend the Task below). Have students investigate how at least two of the different shapes can each represent one-fourth of the whole square.

## Scaffolding and Support

If pairs seem to need more support, consider suggesting that they try using the materials in a specific way. For example:

- Students could use the square inch tiles to cover each of the halves and estimate how many whole square cover each piece to prove that they are the same size.
- Students could cover the rectangle half with the triangle half, cut off the extra part of the triangle and use the piece that was cut off to cover the rest of the rectangle. This can be done by covering the triangle half with the rectangle half as well.
- Students could use sticky notes to cover the rectangle and triangle half pieces. This would help them see that the triangle and rectangle half pieces are two square sticky notes or one square and two half-squares (triangles) and are therefore the same size or area.
Be sure when suggesting materials for students to try that students still try to figure out how the materials could be used without being told directly. For example, consider asking "How might you use the square inch tiles to help you show the triangle and rectangle halves are the same size?"


## Look For

- Students who continue to persevere through the task even if at first they are unsuccessful in solving it. (Resilient)
- Students who investigate the task in multiple or original ways. (Creative)
- Students who apply strategies from previous lessons/parts of this lesson to investigate this task in different ways. (Strategic)


This part of the discussion engages students in Encapsulation because students have to articulate that the different shapes can still represent one-half because they are the same size.

## Evaluate

As pairs are wrapping up their investigation, bring the class back together. Begin the discussion by asking each pair that investigated the shapes that represented one-half how they investigated the problem and what they learned about representing one-half. Some questions that could be asked during this discussion are:

- How do the rectangle and triangle shape both represent onehalf?
- Why is it important that we recognize that one-half can be represented in different ways?

Sample student responses may include:

- They both are half of the square. (Ask students to elaborate on this idea, to prove why.)
- I use the same number of squares/triangles to cover each of them. (Ask students to show an example of this)
- I can cut the rectangle (or triangle) to make it fit on the triangle (or rectangle). (Ask students to show an example of this.)

Have the students who completed the task by investigating how onefourth pieces that are different shapes could represent one-fourth share what they did with the whole class. Encourage them to share examples.

Follow this explanation by the students with this for the whole class to consider - Today we investigated that we can divide a shape into half in different ways and end up with halves that are different shapes, like $\square$ and $\Delta$ Is this ok? Why or why not?

Sample student responses may include:

- Yes, it's ok because the two shapes cover one another (Ask students to show an example of this)
- Yes, it's ok because the squares you start with are the same and they are both broken in two equal pieces.




## Shapes for Extend the Task



## Shapes for Extend the Task



## Shapes for Extend the Task



## Shapes for Extend the Task




## Lesson M2.8

## Skip Counting by 10s

In this lesson, students will be skip counting by 10 s starting at a given number. Students will use the point of view strategy by thinking about how their answers might change based on their starting number. Students should ultimately understand that the digit in the "ones" place will stay the same yet the digit in the tens place will increase by one (10) each time they count on by 10 .

## CCSS.MATH.CONTENT.2.NBT.A. 2

Count within 1000; skip-count by $5 \mathrm{~s}, 10 \mathrm{~s}$, and 100 s.

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively

Time Period: ~60 min
It should be possible to complete this lesson in one ${ }^{\sim} 60$ minute class period. However, to allow students to investigate the tasks and concepts in this lesson fully, it may take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered during the Explain section before moving to the Elaborate/Extend section.

## Materials

10 sets of Two Hands (see end of lesson)
White boards
Whiteboard Markers
Erasers
Base 10 blocks (partners)
Hundreds Chart (partners)
Recess Problem recording sheet (see end of lesson)
Recess Problem (Extend) recording sheet (see end of lesson)
Open Number Line and Target Task -
https://docs.google.com/presentation/d/1BPOY67 f6YyKcrCnCumx4mc5GAJ KHQV4u1TVRBnhUkw/edit?usp=sharing

## Engage

Have students form a seated circle and be sure they can see their classmates. Begin the Engage section by having the teacher hold out 2 closed fists. Ask students, "How many fingers am I holding up?" Listen for students to say
zero. Next, ask the child to the left to hold up all their fingers. Ask the group how many fingers that child is showing (10). Have them keep their 10 fingers showing and ask the next student to hold up all their fingers. Pose the question, "How many fingers are we all holding up now?" (20) (If students only count the fingers one student is holding up, encourage them to notice how many fingers are being held up by everyone together). Repeat this process until everyone is holding up 10 fingers. This will likely go above 100 so will give students the chance to hear counting by 10 s over 100.

Ask any students how they know what number to say next when the next person holds up their fingers. Ask students why counting by 10 s might be useful and in what situations outside the classroom they could use skip counting by 10s. Have multiple students share their ideas with the class.

## Explore

Explain that sometimes when we count by 10s we do not start with zero, but we start with another number. Split the class in 4 groups - each group will begin counting by 10 s from a different number and will show their thinking on their white boards.
Group 1 - start at 2 Group 3 - start at 6
Group 2 - start at 3 Group 4 - start at 9

Have students in the groups pair up and work with a partner. Each partner pair should show how they could count by 10s starting with the given number. Encourage students to use a visual representation such as drawing, hundreds chart, using a number line, using 10-frames, etc. Students should also include a list of the numbers they would say if they were counting by 10 starting with the given number.

As partner pairs complete the task, have them compare their representations and written list of numbers with the other pairs in their group. Students should confirm that they have the same set of numbers that they would count if they counted by 10 s starting with the given number.

## Olook For

- Students who notice the pattern of changing the 10 s place each time they count by 10 (perceptive)
- Students who use multiple representations to show their thinking (resourceful)


## Explain

Gather students to a class discussion space where they can see one another and the board. Have the pairs in each group share the representations they
created for how to skip count by 10s starting with their given number.
Students should explain why they decided to represent skip counting in that way. For example, students who used ten frames might say that they know that there are 10 dots in each ten frame so they can use that to count on. As each pair shares, make a list of the ways that students represent counting by tens, such as, ten frames, number lines, list of numbers, pictures/drawings, etc.

If no student pairs used a number line to represent counting by 10 , this would be a good time to introduce it. Using an open number line, start with one of the given numbers, 2 for example. Then add the "hop", a set of hands below it, and write 10 above it to show counting by 10 . Ask students, what number should be at the point at the end of the hop. Continue this for several rounds.

After all pairs have shared, have the students discuss what they noticed about the representations. This would be a good time to have students talk with a partner before sharing out their ideas to the class. Students should share ideas about what the representations have in common and how they are different. Also ask students to share if seeing the representations that others shared from their point of view helped them to see different ways to represent counting by 10.

OLook For?

- Students who share many or unique ideas about how to represent counting by 10 starting with any number (creative)
- Students who use different materials to represent their thinking (resourceful)
- Students who elaborate on their explanations (communicative)


## Elaborate/Extend

Target Task
Pose the following problem to the students. It can be written on chart paper or projected on the board. (You can substitute the teacher's name in the problem for the name of a teacher in your school building that the students would know.)

Ms. Mindy's class is working to earn extra outdoor recess time. They need to earn 75 points to have more recess. Every day they have the chance to earn 10 points.

As of today, they have 12 points.
How many more days will it take them to reach the goal of 75 points?

This problem is related to the concept of skip-counting by 10s because students can use this as a strategy to determine how many more days it will take them to earn at least 75 points.

Facilitate a discussion about the problem with the students to make sure they understand what the problem is about. Have students work in pairs to solve the problem and record a representation of how they solved it on the Recess Problem recording sheet. Students may use a variety of methods to do so, such as a number line, base 10 blocks, 10 frames, etc.

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Take note as students are working which representations and strategies they used to solve the problem. You will group students to explain their perspective during the Evaluate section according to the strategies they used.

Extend the Task
For students who were observed engaging in one of the high-potential behaviors or who demonstrated advanced understanding of the concept in the Explore or Explain sections of the lesson, the target task can be extended by increasing the complexity of the task in this way -

Ms. Mindy's class is working to earn extra outdoor recess time. They need to earn 75 points to have more recess.
Every day they have the chance to earn 5 points.
As of today, they have 12 points.
How many more days will it take them to reach the goal of 75 points?
The complexity of the task is increasing because now students are skipcounting by 5 s from a number other than 0 or 5 .

Scaffolding and Support
For students who may need some support in solving the problem, consider helping them decide which representation to use. Another representation that could be introduced to support students' thinking is the hundreds chart. Students could start at 12, and count on ten (move down a row) for each day until they are over the goal of 75 points.

OClook For

- Students who develop solution strategies to solve the problem (strategic)
- Students who use different materials to represent their problem


## solving (resourceful)

- Students who show many or unique ways to solve the problem (creative)


## Evaluate

Gather the students back to a class meeting space and engage them in a discussion about how they solved the problem, the representations they used, and how others used their point of view to represent the problem.

Before beginning the discussion, ask groups who did the target and extended task to share their solutions. Write these on the board. If pairs had different answers, tell students the class will determine the correct solution by discussing the strategies and representations.

During the Elaborate/Extend part of the lesson, the teacher documented the ways the groups solved the problem. Call on pairs to share and explain their perspective on the problem according to these strategies. For example, have the pairs that solved it with a number line all share at the same time. Be sure to have all strategies/representations shared/discussed. Consider hanging up the representations of the students' work so others can see it.

If you had students who solved the Extend the Task problem, share that problem with the class and have them share their answer/strategies/representations. Follow this by asking all students to see if there is a relationship between the number of days it took to earn the prize in each of the problems. Ultimately, students could notice that it took the class that earned only 5 points a day double the time (or twice as long) as the class that earned 10 points a day because 10 is double 5 .

Draw students' attention to the different solution strategies and representations used to solve the problem. Ask them to then pick a way that they did not use themselves and explain it to a partner. This way students are looking at the problem from the point of view of how someone else solved it.

Recess Problem Recording Sheet
Name: $\qquad$

Ms. Mindy's class is working to earn extra outdoor recess time.

- They need to earn 75 points to have more recess.
- Every day they have the chance to earn 10 points.

- As of today, they have 12 points.

How many more days will it take them to reach the goal of 75 points if they earn 10 points every day?

## Recess Problem Recording Sheet (Extend)

Name: $\qquad$

Ms. Mindy's class is working to earn extra outdoor recess time.

- They need to earn 75 points to have more recess.
- Every day they have the chance to earn 5 points.

- As of today, they have 12 points.

How many more days will it take them to reach the goal of 75 points if they earn 5 points every day?


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10 points everyday?

100 Chart

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

## Lesson M2.9

## Measurement

In this lesson students will estimate the lengths of various objects and confirm the lengths by measuring using inches and feet as units of measure. Students will use the visualization thinking skill as a strategy to help them estimate if objects are a certain length before measuring.

For more information about measurement concepts, see chapter 12 (p. 271293) in the Math Matters book.

## CCSS.MATH.CONTENT.2.MD.A. 3

Estimate lengths using units of inches, feet, centimeters, and meters.
MP5: Use appropriate tools strategically
MP6: Attend to precision

Time Frame: ~60 minutes
To allow students to investigate the tasks and concepts in this lesson fully, it may take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered during the Explain section before moving to the Elaborate/Extend section.

## Materials

Rulers
1 inch straw pieces
Paper
Glue Sticks
Various Classroom Objects
Sally's Problem (Attached)
Measurement Task Worksheet (Attached)
Measurement Extension Task Worksheet (Attached)

## Engage

Begin the lesson by showing the attached images (labeled "Sally's Problem") and telling the following story:

Sally is so excited for her first day of school! She got a brand new backpack that she can't wait to show all her friends. She wants to

The name "Sally" could also be replaced in the problem with the name of or similar to the students in the class.
bring her favorite book with her, but she's not sure if it will fit in her backpack. How can she figure out if it will fit?

Engage students in a discussion about how Sally can solve her problem. Students may offer ideas like trying to put it in the backpack or some variation of measurement. Lead the class into a discussion about why we use measurement. Questions that might help guide the discussion include these:

- Why is measurement important?
- How can measurement help us in our everyday lives?


## Explore

In this part of the lesson, students will begin exploring the use of the visualization thinking skill as a strategy for estimating lengths. Ask students to close their eyes and picture "one foot." At this point, do not mention measurement or a ruler. Have students share out what they were picturing. Ideas they might share include, their own feet, shoes, a ruler, etc.

Now show students a 1 foot ruler and ask them to share what they know about it. After several students share, put the ruler away and ask them to picture the ruler in their minds. Have a few students share what they are picturing. Then, have students walk around the classroom to find items that they think are about the same length as one foot. Tell students to find at least three items.

## Olook For?

- Students who ask questions about the use of the ruler or the length one foot (curious)
- Students who find objects that are exactly or very close to one foot in length (perceptive)


## Explain

After some time, have the students gather back together and share the items they thought were about one foot in length. Consider having students line up the items next to one another to see if the items are approximately the same length. Students can also share how they estimated that the objects were one foot long. Have students use the ruler to confirm that the items were about one foot.

Next, show students a one inch unit (straw bead). Ask students what they know about an inch. Have several students share ideas and then explain that an inch is a unit of measure. Demonstrate the following:

- Hold up a 1 inch straw unit against the ruler between 0-1 inches. Show how the 1 inch straw is the same as the space between the 0 and 1 on the ruler.
- Ask students to close their eyes and visualize how many 1 inch straw pieces it would take to "fill up" the ruler. Have students share out their ideas.
- Draw a 1 foot line on the board using the ruler. Iterate (repeat the unit) the 1 inch straw pieces to "make a ruler" (start at the beginning of the line, mark the end of one unit, and repeat to the end of the line).
- Line the ruler up under the one foot line the class created. Ask students what they notice about the line and the ruler. Some ideas they may come up with include that the "lines" (units) all match, they are the same length, it takes 12 inches to make a foot, etc.

Review with the student the steps for measuring the length of an object with a ruler. Depending on students' prior knowledge, the teacher could ask students to share the steps for measuring instead.

Facilitate a discussion about the process of measuring using the following questions:

- How can you use visualization to estimate the length of an object?
- Why is it important to line objects up with the ruler before you measure it? What does it mean to "line up" with the ruler? Demonstrate an example and a non-example of this.
- Why is it helpful to be able to measure something?

OLook For?

- Students who explain their reasoning for choosing the objects they did or how they estimated/visualized to determine which objects to choose (communicative)
- Students who explain the importance of measuring correctly (strategic and communicative)
- Students who ask clarifying questions during teaching modeling and class discussion (curious)


## Elaborate/Extend

Target Task
Tell students that they will be doing a task similar to the one they did where they had to visualize one foot and then find an object they thought was about one foot. However, for this task, students will have a handout that lists a different measurement in each row. For each length, students will:

- Visualize the length given. Encourage them to visualize the ruler or straw pieces to help them
- Visualize and try to decide what item in the classroom would be that length.
- Measure the item to find the precise length to the nearest inch.
- Pair students to share their findings. Encourage students to explain their thinking for their measurements.


## Extend the Task

For students who were observed engaging in one of the high-potential behaviors or demonstrated advanced understanding of visualization or measurement, this target task can be extended by having students use the Extend the Task handout instead. This handout includes lengths that are more than one foot. The process for the Extend the Task is the same as the Target Task.

## Scaffolding:

For students struggling with using a ruler to measure the items, provide the 1 inch straw pieces (or a similar manipulative). Remind students to line the inch pieces together without spaces to get an accurate measurement.

## Ontook For

- Students who find varied items for each length given (creative)
- Students who explain their reasoning for each item to their partner (communicative)
- Students who encourage their partners to explain their findings (curious)


## Evaluate

Set up a piece of chart paper (or column on the board) with each of the measurements on the handout (including the lengths on the Extend the Task handout). Have students cut each row of the handout and glue their items to the appropriate chart paper with the appropriate length.

Gather students in a meeting space in the classroom. Go through each measurement and have students share the items they chose and their actual measurements for each. Discuss how students used visualization to help them identify which objects to measure and the differences and similarities between items found. Questions that could be used to guide this discussion include:

- What did you picture/visualize each time to help you think about which item might be that length?
- Why did different people have different items for the same length?
- What did you notice about the actual measurements vs the measurement given?
- What are different ways we can measure that we saw in our lesson today?

Images for Sally's Problem


| Item | Estimate | Actual Measurement |
| :---: | :---: | :---: |
|  |  | Find an item that is about <br> $\mathbf{2}$ inches long. |

Measurement Extend the Task

| Item | Estimate | Actual Measurement |
| :---: | :---: | :---: |
|  | Find an item that is about <br> 14 inches long. | _-_ inches |
|  | 16 inches long. <br> Find an item that is about <br> Find an item that is about <br> 18 inches long. | ind an item that is about <br> 24 inches long. |



## Lesson M3.1

## Lego Arrays

Make sure you have enough lego pieces for students to explore and manipulate materials.

In this lesson, students will explore the relationship between multiplication and division using arrays. This lesson engages students in the Fluency, Originality, Flexibility, and Elaboration Thinking Strategy by having students explain how they developed multiplication and division equations to represent arrays they create. The total number of objects in the arrays are purposeful because they will allow students to represent the total using different arrays. When students are able to come up with many ideas (fluency), combine ideas in new ways or come up with unusual ideas (originality), then categorize and develop their ideas (flexibility and elaboration), they are more able to make inventive or creative connections between ideas.

For information about the interpreting multiplication and division as arrays see the Math Matters book (p. 76-80).

## CCSS.MATH.CONTENT.3.OA.A. 1

Interpret products of whole numbers e.g., interpret $5 \times 7$ as the total number of objects in 5 groups of 7 objects each.

## CCSS.MATH.CONTENT.3.OA.A. 2

Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

## Standards for Mathematical Practice

MP7: Look for and make use of structure.
Time Frame: 2 days
Day 1: Engage, Explore, and Explain 45-60 min
Day 2: Elaborate/ Extend and Evaluate $45-60 \mathrm{~min}$

## Materials

Variety of manipulatives (enough for pairs to have 48 objects)
Lego blocks that are $1 \times \ldots(1 \times 1,1 \times 2,1 \times 3,1 \times 4,1 \times 6)$
Pictures provided in lesson
Sticky notes

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This question engages students in using fluency because they may come up with multiple ideas of how to solve for how many.

## (9)

Evaluating
answers reflects flexibility and elaboration because they must assess and develop ideas.

## Copies of exit ticket

Paper/pencils

## Engage

Present this picture to the class (and have copies available for students, see below).


Pose the question, "How many Lego heads are in the array?" Depending on students' prior knowledge, have a couple of students remind the class what an array is. Allow for student think time and consider having students share solutions with a partner. Facilitate a discussion with the whole class about the number of Lego heads in the picture and how students determined their answers. Begin by asking students to share just the number and record all student answers on a chart or whiteboard, even those that are incorrect (if there are any). If multiple answers are shared, tell students that everyone will evaluate the answers as we share solution strategies.

Then ask the students to explain how they determined the number of Lego heads in the picture. Some student responses might include counting each individual Lego head, repeatedly adding the number of heads in the rows or columns, multiplying the number of rows and columns, etc. The teacher should ask students to record the matching equation for each of their strategies, if applicable. For example, a student who repeatedly added the columns would write $3+3+3+3=12$ or a student who multiplied could write $3 \times 4=12$ (read as 3 rows of 4 heads equals 12 heads) or $4 \times 3=12$ (read as 4 columns of 3 heads equals 12 heads). This discussion will help students begin to create their own understanding of the equations being represented by the array.

## Explore

Have students pair up with a partner for this part of the lesson. Each partner group will work to create an array for the number 24.

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This task engages in originality and fluency because students must show new ways to express their ideas about different ways to represent 24 as an array.

The connection between multiplication and division shows flexibility and elaboration because students must categorize and develop their ideas and connections between ideas.

Students may use any manipulative that the teacher has available, and pairs do not all have to use the same type of manipulative. Tell students that they should write as many multiplication and division equations they can to represent the array they created. Allow plenty of time for students to explore this task. As time allows, students can also create a different array to represent 24.

## OClook For

- Students using both multiplication and division equations instead of just one or the other. (Perceptive)
- Students who develop multiple arrays to represent 24, especially if students use $1 \times 24$ or $24 \times 1$ and explain why this is an array. (Creative)
- Students who use manipulatives from prior lessons or real life to represent the equation (Resourceful)
- Students who continue to work on the task even if unsuccessful the first time. (Resilient)


## Explain

Based on the Explore task, facilitate a discussion of the equations students developed to represent the array of 24 objects. Have students record their equations on a chart or whiteboard as they share out and explain their reasoning for each equation. Be sure that students do not just use numbers in their explanations. Student responses should use the context of the situation when describing how the equation matches the array. For example, a pair of students might create an array of 24 blocks using 3 rows of 8 blocks and represent it using the equation 24=3x8.

During this discussion, also make sure to address the relationship between multiplication and division. This should come from the student ideas that are presented. However, the teacher can also think out loud: "I notice that there are 8 blocks in each row and 3 blocks in each column. I know that I can make this a multiplication equation by arranging them into 8 equal groups of 3 or show it by writing $8 \times 3=24$ or $3+3+3+3+3+3+3+3=24$.

Using the same array, the teacher can explain that the same array of 24 blocks can be represented with division as well. For example,


This task targets originality because students are asked to investigate and create an array.
the teacher could explain that with division we could break the whole set into equal groups. This can be shown with repeated subtraction $24-8-8-8=0$ (this shows 3 groups of 8 being subtracted from 24 with no remainder) or as $24 \div 8=3$ (this equation could be interpreted in two ways - as 24 divided into 8 equal groups of 3 or 24 divided into equal groups of 8 and there are three groups).

Have students describe how the multiplication $3 \times 8=24$ or $8 \times 3=24$ and division $24 \div 8=3$ or $24 \div 8=3$ equations are related. Students might talk with a partner first and then have partners share out with the class.

## OLook For

- Students who clearly articulate their reasoning/justification for how multiplication and division are related. (Communicative)
- Students who share out ideas for their groups and discuss all participants' ideas. (Leadership)


## Elaborate/Extend (Day 2)

Remind students of what occurred during the previous day's lesson.

Introduce the use of Lego bricks as a manipulative for mathematics. The raised dots on the top of the bricks can each be thought of as "1." For example, you might show students one of the $1 \times 6$ pieces and explain that this would be 1 row of 6 dots. Provide pairs of students with Lego blocks.

## Target Task

The task is for students to use the Lego bricks to create their own array using the dots on the blocks. Consider allowing students to select the number they want to work with, but constrain the numbers students can use in their investigation to the following numbers $-48,60,72,84$. Tell students that they should create as many arrays that show the number they selected as possible. An example of a Lego Array for the number 48 is (don't show this to students before they investigate):

Provide sticky notes for students to write equations that match their arrays.

As students are working, observe and ask questions to have students explain their work or strategy. Some sample questions:

- How many arrays can you create to represent this number?
- How else can we represent this number?
- Which of these ideas is most unusual? Why?
- How did you figure out what numbers to use in your array?
- What information is needed to build this array?
- How can you use what you know to build an array of another number?
- How can you build onto your array to represent a bigger number?


## Extend the Task

Students who have previously demonstrated that they can identify multiple arrays can be encouraged to use numbers that are larger, such as 96 or even 108. Even though these numbers have the same number of factors (12) as 84 and 60 , these numbers have more factors that are greater than 12 , making the task more complex.

As students are working the teacher can observe and ask questions to have students explain their work or strategy. Some sample questions include:

- How can you use what you know to build an array of another number?
- How can you build onto your array to represent a bigger number?
- What other lego sizes could you use to build the array, if you had them?

Scaffolding and Support
If pairs seem to need more support, consider ways that they may try using the materials in a specific way. For example:

- Students could be provided with a specific number of Lego bricks of the same size.
- Students could be provided with exactly enough Lego bricks to build an array for the number 48.

As students are working the teacher can observe and ask questions to have students explain their work or strategy. Some sample questions include:

- How would you use these materials provided?
- Should we look at columns or rows first?


## OClook For

- Students who would see the patterns and connect the relationship of multiplication and division with addition and subtraction. (Perceptive)
- Students who create their array and develop multiple equations. (Creative)
- Students who manipulate the Lego bricks to determine all of the possible arrays for the selected number. (Resourceful)
- Students who use different shapes and sizes of legos to create their arrays (originality)


## Evaluate

Have the students conduct a "gallery walk" around the classroom to view the Lego arrays and corresponding equations. The Lego arrays can be left at students' work areas with the equations written next to them on sticky notes. Ask students to observe the array representations that have been created and how the multiplication and division equations match the arrays that are shown.

As an exit ticket, have students revisit the relationship between multiplication and division by asking students to write a response to the following prompt (see end of lesson plan for page to print): Here is an image of a Lego array:


Explain why both $3 \times 5=15$ and $15 \div 3=5$ are equations that could be used to represent the array.

Collect student exit tickets to evaluate their understanding of interpreting products and quotients.

Picture for Engage (copy for students)


Name: $\qquad$

## Exit Ticket

Here is an image of a Lego array:


Explain why both $3 \times 5=15$ and $15 \div 3=5$ are equations that could be used to represent the array.


## Lesson M3.2 A Pizza Party

In this lesson students will be investigating how the arrangement of items in an array can be represented using multiplication and/or division equations using the context of a pizza party. Students will be engaged in the Encapsulation Thinking Skill during this lesson as they are providing explanations for why the multiplication and division equations they developed are accurate representations of the arrays they created and through their writing of a response to the exit ticket prompt.

For information about the interpretation of arrays with multiplication and division see the Math Matters book (p. 76-80).

## CCSS.MATH.CONTENT.3.OA.A. 1

Interpret products of whole numbers e.g., interpret $5 \times 7$ as the total number of objects in 5 groups of 7 objects each.

## CCSS.MATH.CONTENT.3.OA.A. 2

Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

## Standards for Mathematical Practice

MP7: Look for and make use of structure.
Time Frame: ~ 60 minutes
To allow students to investigate the tasks and concepts in this lesson fully, it may take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered the day before during the Explain section of the lesson.

## Materials

$11 \times 17$ paper copy of rectangular party pizza
Anchor chart paper/markers
Rulers, Unifix cubes, circle tokens/markers/chips, etc
Copies of the Exit Ticket


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Students can generate questions about the information that is necessary to solve the problem.

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Students can generate questions about how others solved the problem.

Students should be encouraged to write an equation to match their representation. Look for a variety of equations that represent the ways the students share the pizza- division, multiplication, repeated addition, etc.

## OOLook For

- Students who discover multiple ways to divide the pizza in equal shares. (Creative)


## Explain

Come back together to have students share how they partitioned the pizza into equal shares. Record responses on chart paper. The teacher should keep a record of students who were able to divide the pizza in multiple ways. Ask students to review the responses listed on the chart paper and what questions they might have about how the pizza was divided equally.

Facilitate a discussion of the students' questions and observations. During this time, students might notice or ask about the relationship between multiplication and division to solve a problem (anticipated student responses, e.g. I divided the pizza into 4 rows and 6 pieces in each row because I know $6 \times 4$ is 24 . I divided my pizza into 24 pieces and each person will get 4 because $24 / 6$ is 4 .)

If this idea of the relationship between multiplication and division does not come up from the students' ideas, ask students how they might also represent their partitioned pizza using division. Facilitate a discussion about this between the students.

## OClook For

- Students who provide an extensive explanation and/or were able to discover multiple ways to partition their shape into equal parts. (Communicative)


## Elaborate/Extend <br> Target Task

Ask students to brainstorm some pizza toppings that they enjoy eating on their pizza. Then tell students that they are going to have to create a pizza that spreads the toppings on the pizza so that each piece of pizza has an equal number of pieces of the topping. They should arrange the toppings in straight rows and columns so that they are lined up. The pairs of students should use the pizza they created in the Explore part of the lesson and a manipulative that can be used to represent the pizza topping (limit each group to just one type of topping). Students
 equation (since the arrangement should ultimately be an array).

At this point students may wonder about how many toppings they can use on their pizza. Tell them that this is up to them. However, some students may choose to put more than one topping manipulative on each piece of pizza. Groups that choose this option will need to figure out how to arrange the topping manipulatives to be equal in number on each slice and also be set up in an array.

## Extend the Task

For students who were observed engaging in one of the high-potential behaviors or demonstrated advanced understanding of the concept in the Explore/Explain sections of the lesson, this target task can be extended by asking students investigate how to arrange 4 or more pieces of the topping per piece of pizza. This increases the complexity of the task.

## Scaffolding and Support

If pairs seem to need more support, consider making suggestions or asking scaffolding questions. For example:

- Encourage students to put just one "topping" manipulative on each piece. Their equations to represent the arrangement of the toppings would then match the equation for the number of pieces of pizza.
- Ask students to explain how the equation they write matches the representation with manipulatives.
- Have students explain how the arrays could be written as repeated addition and how this is related to multiplication.


## OClook For

- Students who demonstrate that they are using their knowledge of arrays in creating the arrangement of toppings. (Resourceful)
- Students who develop multiple or unique arrangements on the pizza. (Creative)


## Evaluate

Tell students that they are going to do a gallery walk to look for evidence of arrays and equations for everyone's work. Ask students to think of questions they might ask themselves as they observe others' work. For example, students might say that they can ask what they notice or how the equation matches the representation. Write the questions students generate on the board for all students to refer back to during the gallery walk.

Each group can display their pizza with toppings on a desk/table and the equations they created to represent the arrangement of the toppings. Have the class walk around and make observations about how each of their classmates' groups arranged the pizza toppings. Students should consider some of the questions they generated as well.

After the gallery walk facilitate a discussion of the students' observations related to how the arrangement of the toppings were represented by the equations students wrote. Some prompts that the teacher could use during this part of the lesson are:

- How did increasing the number of toppings on each piece of pizza change the arrangement of the pieces?
- What do we notice about the way that the toppings are arranged on the pizzas? (they are in columns/rows)

After the discussion have each student to complete an exit ticket (see below) and provide their own explanation.

## Party Pizza (copied and enlarged on $11^{\prime \prime} \times 17{ }^{\prime \prime}$ paper)



Name: $\qquad$

## Exit Ticket



Explain why the arrangement of the toppings can be represented with a multiplication or division equation.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$


## Lesson M3.3

## Bake Sale

In this lesson, students will solve two problems related to a Bake Sale theme. Students will engage in the Questioning Thinking Strategy by generating questions about a situation that is presented to them. Information such as the problem question and necessary information will only be presented to students as they identify the need for such information. This represents a more authentic approach to the problem solving process than the ways in which problems are generally presented to students in the mathematics classroom. Additionally, third grade students may be calculating their solutions using different computational procedures, or algorithms.

For more information about and rationale for letting students explore alternative algorithms, see the Math Matters book (p. 43-48).

## CCSS.MATH.CONTENT.3.OA.D. 8

Solve two-step word problems using the four operations.
Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.

Standards for Mathematical Practice
MP1: Make sense of problems and persevere in solving them.

Time Frame: ~60 minutes
To allow students to investigate the tasks and concepts in this lesson fully, it may take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered the day before during the Explain section of the lesson.

Materials
Whiteboards and markers/clear sleeves (1 per student; optional)
Copy/projection of cookie image (below)
Chart paper
Sticky notes

## (B7\%

Students are encouraged to ask questions based on their own curiosity to clarify what is being presented.

## BFy

Students must generate questions they have about the information they need to solve the problem.

## Engage

Begin by telling the class that someone was making cookies for the class and sent you a picture of some of the cookies. Show students the image of the partial plate of cookies (see below). This can be done by projecting the image on a screen, if possible.


Ask students to talk to a partner to come up with questions they have for the baker and/or about the picture. Students might generate questions such as:

- How did you make the cookies?
- How many cookies are on the plate?
- Do we get to eat the cookies?
- Are there enough cookies for everyone? etc.

Note that some of the students' questions may have a mathematical context and others may not. However, if the "how many cookies can each person have?" does not come up, prompt students to think about the cookies in this way so that the question is generated by the students. A good way to scaffold students' thinking in this way might be to ask, "Imagine someone made these cookies for our class, what questions would you have?"

Collect all students' questions and write them on a chart or white board.

## Explore

During the question generating period in the previous part of the lesson, the students may have generated a question such as, "How many cookies can each person have?"

Tell students that this is the question the class is going to investigate. Write the question in a prominent place for all students to see. Ask students if they have enough information to solve this problem, based on what they already know. Students should conclude that they do not have enough information to solve the problem. When this conclusion is

This represents a twostep problem because students first have to determine the number of cookies and then determine how many each student can have.
reached, ask students to talk with a partner to generate questions about the information they need to know to solve the problem.

Have partners share out the questions they develop. Some questions students may develop include the following:

- How many cookies are on the plate?
- How many students in the class?
- Is everyone eating the cookies?
- Does everyone get the same number of cookies? etc.

As students share relevant questions, record the questions on the board or chart paper and provide the relevant information. In response to the questions above, give students the information needed. Because this is a two-step word problem, first tell them how the cookies are arranged on the plate by giving them the details of an array that would represent two times the number of students in the class. For example, if there are 25 students in the class tell them there are 5 rows of 10 cookies). Remind students that everyone will get the same number of cookies. Students should be able to develop a way to determine the number of students in the class, and record this information when students come up with it.

Allow students time to solve the problem with a partner. They may use different strategies to figure out the problem, such as manipulatives, representations/drawings, and/or mathematical equations. Use white boards or a document camera to have partners share out how they solved the problem.

## OClook For

- Students who express and expand their ideas in detailed or organized ways. (Communicative)
- Students who provide clear and exhaustive explanations of their reasoning. (Communicative)
- Students who ask a variety of questions or a question different from their classmates (Creative)
- Students who use multiple strategies to solve the problem (Creative)
- Students who use previously practiced math strategies to solve the problem (Resourceful)


## T

Students are determining the usefulness of questioning as part of the problem solving process in this part of the lesson.


## .

Students are developing metacognitive questions that can be asked throughout the problem solving process.

## Explain

Use this part of the lesson to discuss the Questioning Thinking Strategy with students. Begin by asking students how this task was different from other word problems they usually work on in mathematics. Student responses might include that they had to come up with a question and that they did not have all the information they needed at the beginning of the task.

Ask students what they had to do to develop the problem (they had to ask questions). Facilitate a discussion about this process. You might ask questions like, "How did asking the questions help us to find a solution?" Be sure to elicit responses from multiple students. Students may also build on the ideas of others.

Follow this discussion with the question, "What are some questions we can ask ourselves when solving problems?" Have student pairs record the questions they would like to ask on sticky notes. Students can share out and post their sticky note. Students then will categorize the questions in the following categories: questions for when we first look at a problem, questions for when we're in the middle of solving a problem, and questions for when we think we have a solution (before, during, after problem solving). If the following questions do not come out during the generation of questions by the students, the teacher might "think aloud" that these are some questions the teacher asks while solving problems, and add them to the list - What do I know? What do I need to know? How can I get that information? Does my answer make sense?

Have students come up with the key questions from each category (before, during, after problem solving) to encapsulate the main idea or take away from each section. For example:

- Before - What is the problem? What information do I need?
- During - How can I get that information? What are some ways I can work on this problem? What are some ways I can represent/model this problem?
- After: Does my answer make sense? What new questions do I have?


## Olook For

- Students who generate many and/or varied questions during the discussion. (Creative)
- Students who ask deep and/or complex questions during the discussion. (Curious)


## 5

Students will use questions to guide their development of a two-step word problem.

## Elaborate/Extend

Target Task
Students will work in small groups to create their own two-step word problem. It would be a good idea here to remind students that the cookie problem they solved earlier in the lesson was two steps because first they needed to find out how many cookies were on the plate and then figure out how many each person could have.

Tell students that they can refer to the posted questions from the "Explain" part of the lesson to provide information for the two step word problem they are coming up with. Have students record the word problem they created on a piece of chart paper and hang the chart paper when pairs are done. They should also solve the problem on a separate sheet that can be taped to the back of the chart paper.

## Extend the Task

For students who were observed engaging in one of the high-potential behaviors or demonstrated advanced understanding of the concept in the Explore or Explain sections of the lesson, this target task can be extended by having students increase the complexity of the word problem they are creating. This can be done by encouraging students to:

- Add a third component to their word problem or
- Be sure to use different operations for the steps of the word problem.


## Scaffolding and Support

For students who may need some support determining how to use the questions to guide their development of a word problem, they can focus on forming a one step word problem.

## OOLook For

- Students who represent the problem in ways unique to how others in the class represent it. For example, a student who represents the problem as an equation as part of the response. (Resourceful)
- Students who demonstrate more than one way to solve the problem. (Creative)
- Students who create multiple word problems using various operations. (Creative)


## Evaluate

After all pairs have hung up the chart paper with their word problems

## $\sqrt{7}$

Students should discuss how questioning is helpful throughout the problem solving process.
have students do a gallery walk of all word problems. Before the gallery walk provide students with sticky notes and ask them to write a question they could ask about each of the word problems and post it on the chart paper for that word problem. Students who created the word problems could then read the questions and sort them into questions that would be helpful before, during, and after solving the problem.

Facilitate a discussion with the whole class about how questioning can be used to help them throughout the problem solving process.

Keep the student-generated problems hanging in the classroom for a few days. Over the next few days encourage students to revisit the word problems and solve them. They can check with the group who wrote it or by using the answer sheet on the back of the chart paper.

## Image for Engage part of lesson




## Lesson M3.4

## To Infinity and Beyond with Addition Strategies

In this lesson, students will use the Fluency, Originality, Flexibility, and Elaboration thinking strategy to explore several strategies for determining the value of an addition expression with and without regrouping.

For more information related to how concepts for computation and algorithms (standard and alternative) develop, please see pages 43-45 and 53-54 of the Math Matters book.

## CCSS.MATH.CONTENT.3.NBT.A. 2

Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Standards for Mathematical Practice
MP2 Reason Abstractly and Quantitatively

Note: Consider teaching this lesson before teaching the standard algorithm for multi-digit addition. Then, this lesson could be a good way to assess the prior knowledge students have about multi-digit addition strategies with/without regrouping from second grade.

Time Frame: ~60-80 minutes over 2 days
To allow students to fully investigate the tasks and concepts in this lesson, we recommend teaching over two class periods. A good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered the day before during the Explain section of the lesson.

## Materials

Math Notebooks
Chart Paper
Math Manipulatives available in the classroom
Individual whiteboards
Dry erase markers
Chart paper (stick back or tape to hang up)
Exit tickets
Optional Slide Show: power point

Try to have base ten blocks, whiteboards, tiles, place value discs, etc.

For each unique strategy, have a student share about it. Write yourself a note for who to call on during the discussion.

## 7

Here students are using fluency to generate many ideas and flexibility to explore different ideas.

## Engage

Ask students to model the number 314 in as many ways as they can. They may use paper and pencil as well as any math manipulatives available in the room. Make it known that they can get up and take out manipulatives. Keep in mind, manipulatives being used should be meaningful to the task. (The more manipulatives you use, the longer it will take.)

Engage students in comparing their model with a partner. Students should identify similarities and differences between the ways they modeled the number. After partners share, have several students share their representations of 314 with the whole class. Ask students to explain how all of these different models show the same number. How can we be sure these models all represent 314 ? Be sure to have multiple students contribute ideas to the discussion.

## Explore

Write the expression: " $235+314$ " on chart paper or on the board. Be sure to write the expression horizontally, not vertically (stacked). Ask students to determine the value of the expression (549) using as many ways as they can. Emphasize that the most important part of this task is using multiple strategies, not solving quickly. Encourage students to use any materials in the classroom to help them do so; they may even write different equations or use mental math.

Circulate and note the different strategies students are using to determine the value of the expression to prepare for the upcoming discussion. Look for strategies based on place value such as decomposing into hundreds, tens, and ones, and then adding them back together or using base ten blocks or drawings to model the action of adding two groups of objects together. If students determine the value of the expression in one way, encourage them to determine alternate ways they could represent the same sum with addition. If students need more structure, the teacher may choose to give them a signal to try out a new strategy.

## O Look For

- Students using manipulatives in an unusual way.(e.g., using pattern blocks and assigning a value to each block) (Creative, Resourceful)
- Students who solve the equation quickly and mentally. (Perceptive)
- Students who solve the equation in multiple ways. (Strategic)
- Students who solve the equation in an original way. (Creative)


## 1

Students are using flexibility here because they are being asked to listen to peers and consider other possibilities for solving the same equation.

Partner students together who have similar skills.

## Explain

Engage students in a Four Corners discussion by assigning three corners of the room as the three most popular strategies you saw students use and the fourth corner as "a different strategy." Ask students to move to the corner representing the strategy they felt most comfortable using or their favorite strategy since they may have used multiple strategies. Ask students to talk in their groups about why they like the strategy they choose. Have each group share one sentence about why they like their strategy with the whole class.

Next, have students return to the group meeting area. Have a student from each group show an example of the strategy that their group used. Encourage students to ask each other questions after sharing. Some examples students might ask:

- Why did you choose that strategy?
- How can you prove your answer is correct?
- Was that strategy your first choice? Why?

After students share and discuss their strategies, facilitate a discussion to allow students the opportunity to hear about different strategies. This might help them to consider using a new strategy. Here are some questions you may consider asking during this discussion:

- Which of these strategies is the most efficient and why?
- Which of your classmates' ideas would you try and why?


## Look For

- Students who verbalize their strategy in a concise way. (Communicative)
- Students who ask meaningful questions of their peers. For example: Students might ask: Why did you choose that strategy? How can you prove your answer is correct? Was that strategy your first choice, why? (Curious)
- Students who make connections between multiple strategies. For example: Students might make the connection that using the standard algorithm and using a place value chart are similar ways to solve, just showing it differently (Perceptive)


## Elaborate/Extend (Day 2)

Target Task
Have students work with a partner for this part of the lesson. This task is an extension of the Explore task because it requires regrouping a ten to represent the sum accurately. Remind students of the strategies just


This strategy is being used because they are being asked to determine the value of a new expression in as many ways as they can (fluency) and represent their ideas on a chart (elaboration).
shared and discussed. Then write " $237+415$ " on chart paper or the board (do not stack the numbers). Tell students they may use all the materials that have been available during this lesson, and that they should solve the equation in as many ways as they can. Explain that they also will record/model the strategies they used on a piece of chart paper with their partner.

Provide time for partners to determine the value of the expression. As partners are working, consider asking questions such as - Did you try a new strategy? What made you try that strategy?, How is that strategy similar/different from what you did before? Which strategy worked best for you?

Show students both expressions $(235+314$ and $237+415)$ side by side and ask: What do you notice that is different about the two equations? (They had to regroup for the second equation)

## Extend the Task

For students who were observed engaging in one of the high-potential behaviors or demonstrated advanced understanding of the concept, this target task can be extended by asking students to come up with as many different addition equations as they can that would result in the sum from the target task (652). For example, they might come up with $400+252=652$. Encourage them to work on the equation and note any patterns they observe.

## Scaffolding and Support

If this 3-digit addition expression with regrouping is too complex, provide students with models of two of the strategies using different numbers. Have students pick which strategy works best for them and then apply it to the expression. For example: Show $168+325$ using two strategies such as modeling on the number line, drawing a picture, decomposing into place values, or using base ten blocks. If 3-digit numbers are too complex for a group, give them the expression $37+$ 15.

## OLook For

- Students who try out a new strategy they just learned about. (Resourceful)
- Students who successfully solve the equation in multiple ways. (Strategic)
- Students who continue to come up with additional original strategies. (Creative)


## 7

This strategy is being used because students are evaluating different ways of determining the value of the expression (flexibility).

## Evaluate

Collect students' models on the chart paper and display them in the classroom. Facilitate a discussion about the strategies that students used to determine the value of the expression for this part of the lesson. Questions that could be asked include these:

- What do you notice about all of these charts?
- Which of these strategies would you use the next time you solve an equation?

Have students complete the exit ticket (see below). Use the responses to see which students need more support to use multiple strategies.

Name: $\qquad$
Exit Ticket
Use 2 different strategies to show the total.
$133+448$
$\square$
Bonus: Solve the equation another way.


## Lesson M3.5

The Great Race

In this lesson, students will use the Visualization Thinking Strategy to explore the different ways to represent fractions on a number line. Students should develop the understanding that when fractions are represented on a number line, each line segment should be equal in length. Students will apply visualization to mentally partition a number line into fractional subunits.

For information about the meaning of fractions as measures or fractions on a number line, see the Math Matters book (p. 99-101, 109-111)

## CCSS.MATH.CONTENT.3.NF.A. 2

Understand a fraction as a number on the number line; represent fractions on a number line diagram.

CCSS.MATH.CONTENT.3.NF.A.2.B
Represent a fraction $a / b$ on a number line diagram by marking off $a$ lengths $1 / b$ from 0 . Recognize that the resulting interval has size $a / b$ and that its endpoint locates the number $a / b$ on the number line.

Standards for Mathematical Practice
MP4. Model with mathematics
MP5. Use appropriate tools strategically

Time Frame: ~ 60 minutes
To allow students to investigate the concepts in this lesson fully, it may take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered during the Explain section of the lesson before moving into the Elaborate/Extend section.

## Materials

Sticky notes
White Board or plastic sheet protectors/paper and markers (class set) Manipulatives: square inch tiles, unifix cubes, pattern blocks, any other math materials in your classroom that students might use
$8 \times 11$ inch paper (1 piece per student)
Story of the Tortoise and the Hare - http://read.gov/aesop/025.html
"Number of Hurdles" cards (1 per pair)
Open Number Line Template (1 page per student, so each student gets 4 number lines)
*Note that the women's race is 100 m and the men's race is 110 m . Since when comparing fractions we must start with the same size whole, leave out the distance for the students and have them assume the races are the same distance.

## Engage

Gather students in a space in the classroom that is conducive to discussions and introduce students to the idea of running a race by using the story "Tortoise and Hare." Read the short passage about the Tortoise and the Hare: Library of Congress Aesop Fables (read.gov). Ask students to visualize what is happening in the story. Provide students with the opportunity to share their thoughts. Ask students to visualize the race course as a number line, with the starting line of the race at the zero point and the finish line at the one point, thus representing one whole race distance.

Tell students that there are other types of races too. Show students the videos of the women's hurdles (https://youtu.be/w-6-SSLDrrk) and the men's hurdles (https://youtu.be/jLHg4OX_hSo) races.* Ask students what they notice about these races and how that may differ from the races they visualized after reading the "Tortoise and the Hare."

## Look for?

- Students who can share their ideas clearly or elaborate on their ideas related to visualization. (communicative)
- Students who ask meaningful questions about representing fractions visually. (curious)


## Explore

Continue the discussion by asking students to share what they noticed about the hurdles races. Give students think time and then ask them to share their ideas with a partner. Have multiple students share out what they noticed.

Have students work with a partner to develop a representation (concrete, visual, or symbolic) to show a race that would have only three hurdles. Students can use manipulatives, drawings, or other materials they think would be helpful to represent the race. Allow time for students to brainstorm and develop their ideas. As the pairs of students work, observe to see how students are representing the race. Take note of any pairs that are using a linear or number line representation.

## Olook For

- Students who already are using a number line representation (Perceptive)
- Students who develop representations with different strategies (Strategic)
- Students who apply their learning from the Engage portion of the lesson (Example: Use a strategy their partner mentioned) (Resourceful)


## Explain

Come back together as a class and facilitate a discussion about how the students represented the race. You can use a doc cam or "fish bowl" to have students share their ideas. Be sure to have any students who represented the race in a linear way or as a number line share their ideas with the class.

Regardless of if any students used a linear or number line representation, initiate a discussion of how a number line can be used to represent the race with the three hurdles. Draw an open number line on chart paper/white board for all students to see.

Tell students that this is a number line and we can use it as another way to represent the race. Ask students how we could show the three hurdles on the number line. Students ideas might lead to the number line looking like this to show the three hurdles:


Remind students that the three marks show each of the hurdles and ask students what they notice about the spaces between the hurdles. Students should come up with the idea that the spaces between the hurdles are the same size/length. Then ask students if there is anything missing from the representation. Students should identify that the start and the end of the race is not on the representation. Have students explain what that should look like and add their ideas to the representation. The representation should look something like this:

*Some perceptive students might observe that the actual distance in a hurdles race between the starting line and the first hurdle and the last hurdle and the finish line are different from the distances between hurdles. But for the purposes of this task, students should assume that all are the same.

Ask students if this representation looks like anything that they have used in math before. We want students to bring up the idea of a number line here. Continue the discussion of a number line:

If from here to here (tracing from Start to Finish on the number line with your finger or a pointer) shows the whole race, how can we label where we put the hurdles using numbers?

Give students think time and then ask them to share their ideas with a partner. Have multiple students share out what their ideas of how to label the number line. Student ideas may include that each line segment shows
one-fourth, that the start would be 0 , and that the finish would be 1 . If that comes up you might add to the number line in this way:


Continue using questioning to lead students to label the tick marks (hurdles) with the fractions they represent. Ultimately, the number line should look like this:


Explain to students that we had to partition (divide) the distance from 0 to 1 in equal parts and emphasize that the fraction is not the single tick mark that labels it. The fraction is the distance between the tick marks; each line segment represents one-fourth and the tick mark at the end of each unit tells how many one-fourth units are accumulated from zero.

## Ontook For

- Students who restate their peers' explanation in a different way. (communicative)
- Students who ask questions about others' explanations. (curious)
- Students who clearly articulate their thinking. (communicative)
- Students who relate the concept of the number line to real-life situations. (perceptive)


## Elaborate/Extend <br> Target Task

Tell students that they have been asked by the PE teacher to set up a race with hurdles for their class and the PE teacher needs them to create a
*Note that the actual distance between the starting line and the first hurdle and the last hurdle and the finish line are different. But for the purposes of this task, students should assume they are the same.
number line diagram to show the fractional parts of the race if there are different numbers of hurdles. For example, one group might create a number line fraction representation to show what the race would look like if there were 5 hurdles and another group would create one for 6 hurdles. *Be sure students understand that there is a distance between the starting line and the first hurdle and the last hurdle and the finish line, as well as the distances between each of the hurdles (thus, a race with 4 hurdles would have 5 segments. Also tell students that for our purposes, all the distances between the starting line and the first hurdle, between the hurdles, and from the last hurdle to the finish line are the same length.

Assign students to pairs. Provide each pair a set of the "Number of Hurdles" cards and multiple copies of the open number line template. Explain to students that they should shuffle the pile of Number of Hurdles cards and then flip over the top card. When they have flipped their card, they should visualize a track with a starting line, a finish line, and that number of hurdles. Each student in the pair will create a representation of the hurdles race with a starting line, the hurdles, and the finish line. Once each student in the pair is finished they should compare their number line representations and determine how many fractional parts their race diagram/number line is representing. Ask them to label their representations to show the fractions that are represented. Tell each pair they should complete three different race diagrams to share with the PE teacher (they would complete this task 3 times for different numbers of hurdles).

Extend the Task
For students who demonstrated high potential behaviors related to creating the number line representations and/or their understanding of fraction concepts, modify the target task to increase its complexity. To extend the task, tell the students that the PE teacher also wants to know if there are any race diagrams that will have hurdles in the same places (ex. the diagram with 3,7 , and 11 hurdles will result in number lines that represent $1 / 4,1 / 8$, and $1 / 12$ and would therefore have some equivalent fractions - where the hurdles would be in the same place on the track).

To complete the extended task, students will create three race diagrams/number line representations but give them the sets of cards with $3 / 7 / 11$ hurdles (fractional parts of $1 / 4,1 / 8$, and $1 / 12$ ) or the cards with $2 / 5 / 8 / 11$ hurdles (fractional parts of $1 / 3,1 / 6,1 / 9$, and $1 / 12$ ). Students will then work to compare the fractions among the three race diagrams they complete to identify where there would be hurdles in the same places on each. Have students circle these on each number line representation and discuss how they know these are the same places (represent the same amount of the whole).

## $\boldsymbol{r}$

## Scaffolding and Support

For students who need support or scaffolding to create their fraction number line representations, consider providing cards that have fewer hurdles on them. For example, you can have them do halves (1 hurdle) and fourths (3 hurdles). Students who need support can also be encouraged to use manipulatives to help them make sure the distances between the fractional parts of the number line are equal in length.

## OClook For

- Students who express their ideas about the number line related to the race, starting line and finish line clearly. (communicative)
- Students who use information from the previous tasks with fourths and eighths to help them with the elaborate task. (resourceful)
- Students who use unique strategies to determine how to equally partition the number line. (creative)
- Students who make connections to equivalent fractions independently. (perceptive)


## Evaluate

Label the class white board with the number of hurdles on each card.


Gather students to this part of the classroom and have them tape their race diagrams/number line representations under the corresponding number of hurdles.

Give students time to consider what they notice about the diagrams. Ask them to share some of their observations with a partner. Then, facilitate a discussion about what the students notice about the visual representations on the number lines and record their ideas on a chart paper. Students may make observations such as:

- As the number of hurdles increases, each fractional part of the race decreases.
- As the number of hurdles decreases, each fractional part of the race increases.
- The size of the whole race does not change.

Encourage students who completed the Extend the Task can contribute ideas related to the race diagrams that would have hurdles in the same place. This could be an introduction to equivalent fractions.

| 2 hurdles | 3 hurdles |
| :---: | :---: |
| 4 hurdles | 5 hurdles |
| 6 hurdles | 7 hurdles |





## Lesson M3.6 <br> Two Fair Shares

In this lesson, students will explore fair ways to share an irregular figure in two (halves). Students should come to understand that halves of a figure must be equal in area but not necessarily the same shape. The Encapsulation Thinking Skill is exemplified in the discussion parts of the lesson. Students should precisely explain their reasoning and relate what they did to the idea of partitioning a figure into parts with equal area.

For more information related to fractions as part of a whole, see the Math Matters book (p. 99-105 \& 114-119).

## CCSS.MATH.CONTENT.3.G.A. 2

Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

## Standards for Mathematical Practice

MP3. Construct viable arguments and critique the reasoning of others.

MP5. Use appropriate tools strategically.

Time Frame: ~ 60 minutes
To allow students to investigate the tasks and concepts in this lesson fully, it may take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered the day before during the Explain section of the lesson.

## Materials

Square inch tiles
1 inch grid paper
Scissors
Tangrams
Chart paper/markers
Card stock paper

## Engage

Ask students to brainstorm what they know about fractions. Have students partner talk and then share out some of their ideas and even examples. Record student ideas on a chart paper or white board.

Because this lesson focused on decomposing shapes in halves that are not necessarily the same shape, pose the following questions to the students:

We can show one-half of a square like this $\square$ and like this How can both show one-half of the square?
(You could show this with two sticky notes or draw it for students to see. Just be sure that the two squares are the same size.)

Have students partner talk about this prompt and then ask for multiple students to share their ideas. Encourage students to provide explanations. Ultimately, students should express the understanding that halves of the same size whole can be represented with different shapes.

## Explore

Tell students that usually when they have been learning about fractions as part of a whole they have been using shapes such as squares, rectangles, and circles. However, sometimes we might have to divide other shapes in half.

Show students one of the shapes attached at the end of the lesson and tell them that they are going to have to figure out how to divide or partition the shape in half. Tell students that there are several materials set out that they could use to accomplish the task, and encourage them to select different materials to use. Have the following materials available:

- square inch tiles
- 1 inch grid paper
- scissors
- pencils

Provide time for pairs of students to explore how to partition the shape in half. Observe students' strategies as they are working and encourage students to try different materials if necessary. Note that there are multiple ways that each of these shapes could be divided in half.

## O- Look For

- Students who use an original method to partition the shape in half. This means that they use a method that few, if any, other students in the class are using. (Creative)
- Students who use materials in different ways. (Resourceful)
- Students who use different materials to investigate the task. (Strategic)
- Students who persevere when initial attempts at partitioning the shape are not successful. (Resilient)


## Explain

Come back together to have students share how they partitioned the shape. Use a doc cam or "fish bowl" to have partners share how they partitioned the shape. When students explain their process be sure to ask them to justify how they know that there are now two equal halves of the whole. This is especially important because the two halves may not be the same shape.

## O Look For

- Students who elaborate on their explanation of why the way they divided their shape represents breaking it in half. (Communicative)
- Students who explain unique ways of determining how to partition the shape in half. (Creative)


## Elaborate/Extend

Target Task
Introduce students to the tangrams. If this is something they are unfamiliar with, you will want to provide time for students to explore the materials prior to beginning the task.

* Note that any new figure students create from tangrams can be covered

Students are engaged in the Encapsulation thinking skill during this discussion because they are starting to think about and try to explain what it means to divide an irregular figure in half.

Tell students that they should select 3 or 4 tangram pieces and use those 3-4 pieces to create a new figure on top of a piece of card stock paper. Limit students, at this point, to using only one triangle of each size (small or large). Students should trace the outline of the new figure on the paper in pencil first and then in black marker to show the outline clearly. It may also be helpful for students to cut out the new figure.

Ask students then to show how to partition their new figure in half. Alternately, students can trade figures and partition their partner's figure in half. Encourage them to talk with partners about how they
by the small triangle from the tangram set. Students could use this as a way to help them find half of the new figure.

## 为然:

This discussion engages students in the Encapsulation thinking skill because students have to articulate that shapes can be partitioned in half multiple ways and/or that the halves might have different shapes but still
are partitioning the figures and how they know the partition is creating two halves. Through this task, students will extend their knowledge of how to show ways the shape can be partitioned in half that they were developing in the explore part of the lesson. *(See side comment)

## Extend the Task

For students who were observed engaging in one of the highpotential behaviors or who demonstrated advanced understanding of the concept in the Explore/Explain sections of the lesson, this target task can be extended in the following ways as students complete the Target Task:

- Ask students if their observations can be applied in the same way when working with fourths. Provide them with the opportunity to investigate this.
- Ask students if their multiple/original methods could be used for other figures as well. Provide students with the opportunity to create new figures with more tangram pieces and to determine how to divide that figure in half.


## Scaffolding and Support

If using the tangram pieces is too complex, provide students with 15-30 square inch tiles to use to create their new figure. Students can assemble the tiles, trace them, and complete the same task, but with squares instead of triangles as the unit.

## O- Look For

- Students who develop the idea on their own that all of the shapes in the tangram set can be composed from the smallest triangle in the set. (Perceptive)
- Student who develop multiple or original ways to divide their figure in half. (Creative)


## Evaluate

As pairs are wrapping up their investigation, bring the class back together. Ask each pair to prove that the way they partitioned their shape shows halves.

After all pairs have shared their ideas, pose this prompt: Suppose one of your friends was not in class today. Explain what you learned about partitioning a shape in halves. Provide think time and have students discuss ideas with a partner. Solicit ideas from multiple students and be sure to have them elaborate on explanations and provide examples.
represent half because they are the same size.

Students' responses might include the various ways they divided their shapes in half. Students' responses also might include some of the following ideas:

- We can break the shape apart and put the pieces back together to make each half have equal area. (Ask students to show an example of this.)
- We can measure the area of each half and if they are the same then they are each half of the whole. (Ask students to show an example of this.)
- We can determine how many small triangles fit in the shape and count out half. (Ask students to show an example of this.)

Record students' ideas on chart paper.

## Shape for Explore Task

2 in





## Lesson M3.7 What's My Pattern?

In this lesson, students will engage in the point of view thinking strategy to recognize and explain patterns within sequences of numbers. Point of View is a creative and critical thinking strategy that helps students analyze how different people look at the same situation, in this case patterns. Students will generate their own patterns, determine the patterns of others, and explain their thinking.

For more information about patterns and related algebraic concepts, see chapter 9 (p. 198-219) in the Math Matters book.

## CCSS.MATH.CONTENT.3.OA.D. 9

Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

MP7: Look for and make use of structure

Time Period: ~60 min
It should be possible to complete this lesson in one ${ }^{\sim} 60$ minute class period. However, to allow students to investigate the tasks and concepts in this lesson fully, it may take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered during the Explain section before moving to the Elaborate/Extend section.

## Materials

Chart Paper
Sticky Notes ~ 12 per student

## Engage

Gather students to a class discussion space and ask, "What is a pattern?" Consider having students talk with a partner before sharing out with the class. Have many students share their ideas as part of the discussion. If necessary these questions can prompt students to think about patterns further:

- Where do we see patterns?
- Are there any patterns in the classroom? In nature? At home?
- What are some examples of patterns? What's similar and different about some of our examples of patterns?

Then prompt students to think about where they see patterns in math. Questions that could guide this discussion include: Where do we see patterns in math? Did you notice patterns when you were working with shapes, addition, or subtraction?

## Explore

Write each of the following three patterns on different sheets of chart paper and hang each pattern in a different part of the classroom

YY RR YY RR
15, 21, 27, 33, 39
$2,3,5,9,17,33$
Split the class into three groups and have each group stand near one of the patterns. Ask all groups to look at their pattern and jot down on sticky notes what they notice about the pattern. Students can add the sticky notes with their ideas to the chart paper. Students may write what they think comes next, but also ask students what else they notice about the patterns. Give students a few minutes to complete the task, then have groups rotate through the other two patterns to repeat the task.

## OLook For?

- Students who share multiple ideas about the patterns (creative)
- Students who clearly explain their ideas about the patterns (communicative)


## Explain

Gather students back together at a class discussion space. For each of the patterns, display the chart with the pattern and the students' sticky notes. Have students group sticky notes with similar ideas together and discuss the various ideas that students shared about each of the patterns. If students discuss how they determined the next in the pattern, be sure to have them explain their reasoning. A point you want students to emphasize the repeating nature of a pattern. For example, the $2,3,5,9,17,33$ pattern is a two-step rule - each term is multiplied by two and then 1 is subtracted to get the next term [ 3 comes after 2 in the pattern because ( $2 \times 2$ ) - $1=3$, then ( $3 x$ 2) $-1=5$, and so on].

To engage students in the point of view thinking strategy for each pattern, ask students if they see an idea on a sticky note that they did not think of themselves. Ask to consider how the person who came up with that idea might have looked at the pattern differently.

## OLook For?

- Students who clearly articulate their explanation for the pattern (communicative)
- Students who identified the patterns and were able to make the connection to multiplication (perceptive)
- Students who develop multiple rules to identify the pattern (creative)


## Elaborate/Extend

Target Task
For the first part of the task students will work independently, then they will work with a partner. Explain that each student will create their own number pattern, and that they should use numbers between 20 and 100 to create their pattern.

Have each student work to develop their pattern and record the numbers in the pattern on the Pattern Recording Sheet. They should also record the rule for their pattern on the back of the sheet.

As students complete their patterns, pair them with a partner and have them exchange patterns. Each person in the pair should work to identify the
 When students come up with different rules that work for the same pattern, this is a good opportunity for the point of view thinking strategy.
pattern, add on the next three numbers in the pattern series, and explain the pattern from their point of view. Note that there may be different ways for students to explain the same pattern. Have students discuss how different rules could result in the same pattern when it arises.

Questions to ask as students are working include:

- How can you determine if your partner's pattern is correct?
- What other patterns can you make using the same numbers?
- Is there another rule that matches your pattern?


## Extend the Task

For students who were observed engaging in one of the high-potential behaviors or who demonstrated advanced understanding of the concept in the Explore or Explain parts of the lesson, the target task can be extended in a few different ways. Select one or more of these options to increase the complexity of the task:

- Create a pattern in which the rule has two or more steps
- Use numbers greater than 100 to create the pattern


## Scaffold and Support

For students who may need some support in developing their pattern the task can be modified in these ways:

- Prompt students to create a pattern using numbers less than 20
- Provide students with a number line to show their pattern
- Provide students with a hundreds chart to show their pattern
- Ask questions such as, "How are these numbers related?" or "What number do you think would come next?"

OLook For:

- Students who can communicate the thinking of others effectively (communicative)
- Students who ask questions about students' thinking (curious)
- Students who develop original ideas, different from their classmates (creative)
- Students who continue to work on the task even if unsuccessful or continue to work to come up with alternative responses (resilient)


## Evaluate

Gather the class together in a class meeting spot and have students bring their Pattern Recording Sheets with them. Have each student share the rule for their pattern. As students share, make columns on the board, write the pattern rules at the top of each column, and tape the students' Pattern Recording Sheets under the rule for their pattern. After all students' patterns are posted, have students examine the patterns that all have the same rule (are in the same column). Chances are students used different numbers to show the same pattern. Engage students in a discussion about how different people created different patterns using the same rule from their own point of view.

Name: $\qquad$

Write your number pattern below.

Name: $\qquad$

## Exit Ticket

What number is missing from the following pattern? Explain how you determined the missing number.

| 24 | $?$ | 40 | 48 | 56 | 64 |
| :--- | :--- | :--- | :--- | :--- | :--- |

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

What's My Pattern? Exit Ticket
Name: $\qquad$

## Exit Ticket

What number is missing from the following pattern? Explain how you determined the missing number.

| 24 | $?$ | 40 | 48 | 56 | 64 |
| :--- | :--- | :--- | :--- | :--- | :--- |

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$


## Lesson M3.8 Making Cookies Equivalently

In this lesson, students will explore equivalent fractions and engage in the decisions and outcomes thinking strategy. Students will use measuring cups to determine which fractions are equivalent. After that students will look at a cookie recipe and need to convert the recipe using only one measuring cup. Students will need to explain the decision that they have made and what the outcome was based on that decision.

For more information about measurement concepts, see chapter 5 (p. 114119) in the Math Matters book.

## CCSS.MATH.CONTENT.3.NF.A.3.B

Recognize and generate simple equivalent fractions, e.g., $1 / 2=2 / 4,4 / 6=$ $2 / 3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

Standards for Mathematical Practice
MP7 Look for and make use of structure
MP2 Reason abstractly and quantitatively

Time Frame: ~60 minutes
To allow students to investigate the tasks and concepts in this lesson fully, it may take more than one class period. If the lesson will extend across two class periods, a good place to pause the lesson is after the Explain section. When restarting the lesson, be sure to start with a brief review of what students discovered during the Explain section before moving to the Elaborate/Extend section.

## Materials:

How many scoops? Recording Sheet
Cookie Recipe sheet
8 sets measuring cups and spoons (remove the one-third, two-thirds, and three-fourths scoops from the set)
Class set of fraction cubes
Receptacles for water

## Engage

Gather the students in a class meeting space and share this picture of a cake to the students:


Ask the students - Would you prefer $\frac{1}{2}$ of this cake or $\frac{3}{6}$ of this cake? Have students who would prefer one-half of the cake stand on one side of the meeting space and have students who would prefer three-sixths of the cake stand on the other side. Students who are undecided can stand between the groups. Ask several students from each group to share their ideas about why they would prefer one-half of the cake, three-sixths of the cake, or are undecided.

Some students may bring up the idea that one-half and three-sixths of the cake are the same amount. However, if they do not, introduce this idea, perhaps in a way such as this "Another student in Ms./Mr. (insert name of other 3rd grade teacher)'s class said that they would be fine with either onehalf or three-sixths of the cake because they are the same. What do you think about that?"

Continue to facilitate this discussion by asking, "How can one-half and threesixths be the same amount of the cake?" Have students talk with a partner and then several students can share out their ideas.

Introduce students to the term - Equivalent Fractions. As a class, make a definition of an equivalent fraction. Write this on the board or on an anchor chart where it can be referred to again at the end of the lesson.

OLook For?

- Students who ask a variety of questions or a question different from their classmates (curious)


## Explore

For this part of the lesson, students will work in groups of 3 or 4 . Introduce students to the measuring cups. Invite students to talk about what these are used for and give examples of when they have used them.

Distribute the How Many Scoops? Recording Sheet. Have each group use water and the one-fourth and one-eighth cup scoops to determine the equivalent fractions for 1 whole, one-half, and one-fourth (for the oneeighth scoop). They will use the water to determine how many one-fourth and one-eighth scoops it takes to equal the other amounts and record this on the sheet.

## OClook For

- Students who express and expand their ideas in detailed or organized ways (communicative)
- Students who use multiple strategies to solve the problem (creative)


## Explain

Gather students together in a class meeting space to go over their findings. Consider projecting a blank How many scoops? table for the one-fourth and one-eighth trials. Have groups share out their ideas to complete both tables. Be sure to ask questions such as "How do you know?" and "How can we show this another way?"

Revisit the definition for equivalent fraction that was developed at the beginning of the lesson. Ask students if there is anything they want to revise in the definition that they created.

## Olook For

- Students who provide clear explanations of and/or elaborate on their reasoning (Communicative)


## Elaborate/Extend

Target Task
Explain to students that today they are going to take a look at a recipe for cookies. Consider projecting this for all students to see. Ask students what they notice about the recipe. Then direct their attention to the units of measurement being used in the recipe. Many of the ingredients need whole cups or one-half cups to measure them.

Explain to students that "Oh No!" the 1 cup and $\frac{1}{2}$ cup scoops have gone missing and that they will need to rewrite the recipe using either $\frac{1}{4}$ cup or $\frac{1}{8}$ cup measurements. It will be up to each pair to decide which one they want to use. Once they decide they should determine the equivalent fraction for each ingredient listed on the Cookie Recipe sheet and then record a way that they can prove that the two fractions are equivalent. Groups can choose different manipulatives such as the fraction cubes to prove this.

|  | Extend the Task |
| :---: | :---: |
| 『 | For students who were observed engaging in one of the high-potential behaviors or demonstrated advanced understanding of the concept in the |
| $\bigcirc$ | Explore or Explain sections of the lesson, this target task can be extended by having students use the tablespoon ( $\frac{1}{16}$ of a cup) as their new measurement. However, do not tell them that it is $\frac{1}{16}$ cup - they should first work to figure this out. Students completing this extended task can use the same recording sheet, but write in $\frac{1}{16}$ above the table. |
| 」 | Scaffolding and Support <br> For students who may need some support determining the equivalent fractions, remind students of the manipulatives that they can use to do so. Students may also use water, the 1 cup scoop, and the $\frac{1}{2}$ cup scoop to determine the equivalent fractions. |
|  | OLook For? <br> - Students using manipulatives in an unusual way. For example: Students may use fraction cubes to show various fraction comparisons. (creative) <br> - Students who show the equivalencies in multiple ways (strategic) <br> - Students who identify a pattern in the equivalencies (perceptive) <br> - Students who solve the equivalencies in an original way (creative) |

## Evaluate

When groups have finished, have each group choose one of their ingredients to present. The group should share what fraction cup they used, and what strategy they used to prove the outcome that they used to ensure the fraction is equivalent.

Facilitate a discussion about how the recipe changed when using a onefourth or one-eighth (or one-sixteenth) cup scoop. Would this change the outcome of how much batter they were able to make? At this point students should recognize that the amount of batter they would make would be the same because the amount of each ingredient they used was equivalent to the original. If students are unsure of this, they could use water to show the total amount used for the original and equivalent fraction recipe. The total amount of water for each would be the same.

Revisit the definition for equivalent fraction that was developed at the beginning of the lesson. Ask students if there is anything they want to revise in the definition that they created.

Name: $\qquad$
Use water to figure out how many $\frac{1}{4}$ cup scoops are equivalent to the other scoops.

| Measuring cup | Tally Marks for Number of <br> $\frac{1}{4}$ Scoops | Write the Equivalent Fraction |
| :---: | :---: | :---: |
| 1 cup |  | $\mathbf{1}$ whole is the same as |
| $\frac{1}{2}$ cup |  | $\frac{1}{2}$ is the same as |

Use water to figure out how many $\frac{1}{8}$ cup scoops are equivalent to the other scoops.

| Measuring cup | Tally Marks for Number of <br> $\frac{1}{8}$ Scoops |  |
| :---: | :--- | :--- |
| 1 cup |  | 1 whole is the same as |
| $\frac{1}{2}$ cup |  | $\frac{1}{2}$ is the same as |
| $\frac{1}{4}$ cup |  | $\frac{1}{4}$ is the same as |

Name: $\qquad$

## Chocolate Chip Cookie Recipe:

| Ingredients: |  |
| :--- | :--- |
| 1 cup butter | 2 egg |
| $1 / 2$ cup sugar | 2 teaspoons vanilla extract |
| 1 cup brown sugar | 1 teaspoon baking soda |
| 2 cups all purpose flour | 1 teaspoon salt |
| $3 / 4$ cup chocolate chips |  |
| Directions: |  |
| 1. In a stand mixer, cream butter and sugar until light and fluffy. |  |
| 2. Add egg and vanilla. Beat until just combined. |  |
| 3. Add baking soda and salt. |  |
| 4. Mix in flour a little at a time. |  |
| 5. Stir in the chocolate chips. |  |
| Bake $10-13$ min. At 350 degrees F. Makes 16 cookies |  |



Rewrite the ingredients using an equivalent fraction to show how you could use the $\frac{1}{4}$ or $\frac{1}{8}$ cup scoop. Circle which one you decide to use.

| Ingredient | Measurement |  | Equivalent Fraction and Prove it: |
| :---: | :---: | :---: | :---: |
| Butter | 1 Cup |  |  |
| Sugar |  |  |  |
|  |  |  |  |
|  |  |  |  |


| Ingredient | Measurement |  |
| :---: | :---: | :---: | :---: |
| Brown Sugar | 1 Cup |  |
| Flour | 2 Cups |  |
| Chocolate |  |  |

