is a binary operation that equals $\frac{1}{3}$ and the exponent (power of 2) is a unitary operation: $(\frac{1}{3})^2 = \frac{1}{9}$. The parenthesis, a grouping symbol, lets us know which operation to perform first. Likewise, the addition $9 + {}^{-}7$ is a binary operation which is completed first. The absolute value is then taken on the result, 2.

Another important group of symbols are classified as *relational symbols*. Relational or relation symbols establish a relationship between two numbers, two number sentences, or two variable expressions. Some common relational symbols are $=$, $\neq$, $\leq$, $\geq$, $<$, and $>$. The relationship is either true ($10 = 10$) or false ($5 > 8$), though in the case of open sentences ($9 + 3 = \square + 4$) we always try to find a value to make the sentence true. What can be confusing is that on either side of the relational sign there may be operations that must be completed in order to evaluate whether or not the relationship is true or false. Examine the following and consider whether the relationships that are being established are true or false.

$6 \neq 14 \div 2$	True, 6 does not equal $14 \div 2$, or 7
$18 \geq 19$	False, 18 is not greater than or equal to 19
$5 + 7 = 2 \times 6$	True, $5 + 7$ equals 12 and 2×6 equals 12, $12 = 12$
$\square < \square + 3$	True, any number is less than that same number plus 3

Students need to examine many different number sentences such as the first three examples above and evaluate whether they are true or false. Teachers can then ask students to change the sentences, making true ones false and false ones true. Or students might be asked to group number sentences and explain why they placed different sentences together. Asking students to discuss the relationship between the quantities on each side of a relational symbol will help them interpret these symbols correctly. Open sentences in which one or more variables are represented are especially problematic for students. Research has shown that in open sentences like $8 + 4 = \square + 5$, students in grades 1 through 6 overwhelmingly think $\square$ equals either 12 or 17 (Falkner, Levi, and Carpenter 1999). They add the first two or all three numbers and do not interpret the equal sign as a symbol that establishes a relationship between the quantities on either side of it. This important idea is explored in more detail in the next section.

3. Equality

Another fundamental idea of algebra is *equality*. Equality is indicated by the equal sign and can be modeled by thinking of a level balance scale. Why is equality important for students to understand? First, the idea that two mathematical expressions can have the same value is at the heart of developing number sense. For example, we want students to realize that there are many ways to represent the same product ($9 \times 4 = 2 \times 3 \times 6$). We want students to use what they know about the composition of numbers to help them remember number facts and form equivalent statements ($7 + 6 = 6 + 6 + 1$ and $7 + 6 = 7 + 7 - 1$). Understanding these number sentences and the relationships expressed by them is linked to the correct interpretation of the equal symbol.

The second reason for understanding the concept of equality is that research has shown that lack of this understanding is one of the major stumbling blocks for students when solving algebraic equations. To solve $3x + 2 = 14$, we add $^{-}2$ to both sides of the equal sign, thus maintaining balance or equality ($3x + 2 + {^{-}2} = 14 + {^{-}2}$, or $3x = 12$). The next step in the solution process involves multiplying both $3x$ and 12 by $\frac{1}{3}$, again because performing the same operation on equivalent expressions means they will remain equivalent ($(\frac{1}{3})3x = 12(\frac{1}{3})$, or $x = 4$). If students do not understand the idea of equality of expressions, they may perform computations on one rather than on both sides of an equal sign.

Yet the concept of equality is not easy for students and many do not correctly interpret the equal symbol. For example, often students think that the equal sign is a symbol that tells them to do something (such as subtract or multiply) rather than a symbol that represents equal values or balance. Students see an equation such as $6 + 4 = \square + 6$ and assume 10 is the answer because they have completed the addition on the left: to them the equal sign means "fill in the answer." Other children cannot make sense of $7 = 9 - 2$ because the operation symbol is to the right of the equal sign rather than the left. When students make these types of mistakes, we need to ask them to explain their thinking and to share with us what the equal sign means to them. However, telling students that the equal sign is, by convention, the symbol that lets us know that quantities are equal is not sufficient to clear up their misconceptions. We need to include activities and discussions in our instruction that focus on understanding equivalent values. Exploring whether number sentences are true or false is one activity that helps build understanding with number sense.

Another activity that supports understanding equality is to use balance scales. The balance scale is a visual model for the equality relationship. Most students intuitively understand that a balanced scale remains balanced if equal amounts are added to or subtracted from both sides of the scale. Balance scale problems can be used to investigate equivalence and to prepare students informally for symbolic representation and more abstract solution techniques.

Activity

Balance Scales

Objective: explore the concept of equality using a balance scale.

Solve the balance scale problems on page 197. On each of the balance scales, assume that the same shapes represent the same weights. In each problem, use the information from the balanced scales A and B to figure out what's needed to balance scale C.

Things to Think About

If you have never examined problems like these, there are a number of general principles you have to consider. First, a level scale implies that the quantities on one pan are equivalent in weight to the quantities on the other pan. Second, we can modify both sides of the balance scale and maintain equality using either additive reasoning—removing (or adding) the same amount from (to) both sides of the scale—or multiplicative reasoning—multiplying or dividing both sides by the same factor. (If two cylinders balance six spheres, then one cylinder is equivalent in weight to three spheres). Third, we can replace objects of equal weight. These